Online Appendix:
When Uncertainty Blows in The Orchard

Not For Publication

This Appendix is divided into five sub-appendices. The first section gives additional explanation to data sources and construction. Section 2 presents some technical proofs which have been omitted in the paper. Section 3 contains additional Tables, Section 4 carries out some additional robustness checks and finally section 5 explains the concept of dispersion trading.

Contents

1 Data Appendix

1.1 Construction of Proxies of Volatility and Correlation Risk Premia

Option data is from Optionmetrics and we apply a number of data filters to circumvent the problem of large outliers. First, we eliminate prices that violate arbitrage bounds, i.e., call prices are required not to fall outside the interval $\left(Se^{-rd} - Ke^{-\tau r}, Se^{-\tau d}\right)$, where S is the price of the underlying asset, K is the strike price, d is the dividend yield, r is the risk-free rate, and τ is the time to maturity. Second, we eliminate all observations for which (i) the ask is lower than the bid price, (ii) the bid is equal to zero, or (iii) the spread is lower than the minimum tick size (equal to USD 0.05 for options trading below USD 3 and USD 0.10 in any other case). To mitigate the impact of stale quotes we eliminate from the sample all observations for which both the bid and the ask are equal to the one on the previous day. We focus on short-term options, which are known to be the most liquid, with a time to maturity between 14 and 31 days.

Volatility and Correlation Risk Premia:
We define the variance risk premium from day t to T as follows:

$$VRP(t) \equiv IV(t) - RV(t+1),$$

where $IV(t)$ is the one month at-the-money implied volatility and $RV(t+1)$ the one month realized volatility.

Implied Volatility:
To get an estimate of the risk-neutral implied variance, we follow Demeterfi, Derman, Kamal, and Zhou (1999) and Britten-Jones and Neuberger (2000). They show that if the underlying asset price is continuous, the risk-neutral expectation of total return variance is defined as an integral of option prices over an infinite range of strike prices. Since in practice the number of traded options for any underlying asset is finite, the available strike price series is a finite sequence. Suppose the available strike prices of the call options belong to $\left[\underline{K}^{call}, \overline{K}^{call}\right]$, where $\overline{K}^{call} \geq \underline{K}^{call} \geq 0$. As shown in Jiang and Tian (2005), a truncated version of the integral over the infinite range of strike prices can be used to evaluate the model-free implied volatility. Denote $C(T, K)$ the spot call price with strike price K expiring at time T and F_t the forward price. Using the expression of Britten-Jones and Neuberger (2000) for the model-free implied variance, we use the trapezoidal rule to numerically calculate the integral:

$$2\int_{\underline{K}^{call}}^{\overline{K}^{call}} \frac{C(T,K) - \max(0, F_t - K)}{K^2} dK \approx \frac{\overline{K}^{call} - \underline{K}^{call}}{m} \sum_{i=1}^{m} \left[g_{t,T}\left(K_i^{call}\right) + g_{t,T}\left(K_{i-1}^{call}\right)\right].$$

where

$$g_{t,T}\left(K_i^{call}\right) = \frac{C\left(T, K_i^{call}\right) - \max\left(0, F_t - K_i^{call}\right)}{\left(K_i^{call}\right)^2}, \tag{1}$$

and K_i^{call} is the i^{th} largest strike price for the call option. To implement the trapezoidal rule, we need the option prices $C\left(T, K_i^{call}\right)$, for $i = 1, \ldots, m$. Since some of these prices are not available, we apply a cubic spline interpolation method as proposed in Forsythe, Malcolm, and Moler (1977) to obtain the missing values.[1]

Let

$$MIV_t^{(\tau)} = \frac{\overline{K}^{call} - \underline{K}^{call}}{m} \sum_{i=1}^{m} \left[g_{t,T}\left(K_i^{call}\right) + g_{t,T}\left(K_{i-1}^{call}\right)\right], \tag{2}$$

[1] Jiang and Tian (2005) take a different approach: They first calculate the implied volatilities of available options with the Black and Scholes formula, and then use the interpolation method to obtain the Black and Scholes implied volatilities of the unavailable options. Using these implied volatilities, they use the Black and Scholes formula again to obtain the continuum of option prices. They claim that their method can avoid the nonlinearity problem in the option prices. However, we find a direct use of the interpolation method on the option prices to be more robust.

where $\tau = T - t$ denotes the time horizon or time to maturity. As mentioned above, we replace F_t in equation (1) by the futures price.

Realized Volatility:
Realized volatility is the square root from the sum of daily squared returns over a window of 21 trading days.

Correlation Risk Premium:
The correlation risk premium can be measured using correlation swaps. A swap buyer pays the implied correlation $SC_{t,T}$ at maturity T and receives the average realized correlation $RC_{t,T}$ in a basket of stocks, where $SC_{t,T} = E_t^{\mathbb{Q}}[RC_{t,T}]$, with $\mathbb{Q}$ the relevant risk-neutral probability. The payoff of a (normalized) long correlation swap is $CR_{t,T} = RC_{t,T} - SC_{t,T}$ and its expected value is the correlation risk premium: $CRP_{t,T} = E_t^{\mathbb{Q}}[RC_{t,T}] - E_t^{\mathbb{P}}[RC_{t,T}]$.

Observable proxies for $SC_{t,T}$ can be inferred from the cross-section of index and individual variance swaps. A long variance swap pays implied variance of a given asset at maturity T, i.e., the variance swap rate $SV_{t,T}$, and receives the average realized variance $RV_{t,T}$. The payoff of a (normalized) long variance swap is $VR_{t,T} = RV_{t,T} - E_t^{\mathbb{Q}}[RV_{t,T}]$ and its expected value measures the variance risk premium: $VRP_{t,T} := SV_{t,T} - E_t^{\mathbb{P}}[RV_{t,T}]$. We follow Driessen, Maenhout, and Vilkov (2009) and synthesize the implied correlation $IC_{t,T}$ using the relation:

$$IC_{t,T} \approx \frac{SV_{t,T}^I - \sum_{i=1}^m w_i^2 SV_{t,T}^i}{\sum_{i \neq j} w_i w_j \sqrt{SV_{t,T}^i SV_{t,T}^j}} \,, \tag{3}$$

where $SV_{t,T}^I$ and $SV_{t,T}^i$, $i = 1, \ldots, n$ are index and individual variance swap rates, respectively, and w_i is the market capitalization of stock i at time t. Index and individual variance swap rates are synthesized from (listed) plain vanilla option prices.[2]

Market Volatility: We calculate market volatility from CRSP and OptionMetrics as the 21-day realized volatility of daily returns.

Skewness: As a proxy for risk neutral skewness we use the difference between the implied volatility of a put with 0.92 strike-to-spot ratio (or the closest available) and the implied volatility of an at-the-money put, dividend by the difference in strike-to-spot ratios.

Macro Factor: We estimate a latent macro factor using dynamic factor analysis applied to the time-series of industrial production, housing start number, producer price index, non-farm employment, and S&P500 P/E ratio. We retrieve S&P 500 price-earnings data from the S&P webpage and the other macro variables from FRED.

Liquidity: We use two different measures of liquidity. We control for stock and bond market liquidity effects. Data is from CRSP and OptionMetrics. We also account for equity market liquidity using the Pástor and Stambaugh (2003) measure of liquidity, which is available from WRDS.

[2]Under the assumption of no arbitrage and a continuous swap rate process, the following relation is exact (see, e.g., Carr and Madan, 1998, Britten-Jones and Neuberger, 2000 and Carr and Wu, 2009):

$$SV_{t,T} = E_t^{\mathbb{Q}}\left(RV_{t,T}\right) = \frac{2}{(T-t)\,B(t,T)} \int_0^\infty \frac{P(K,T)}{K^2} dK, \tag{4}$$

where $B(t,T)$ is the price of a zero coupon bond with maturity T and $P(K,T)$ the price of an out-of-the-money put option with strike K and maturity T. We define the 30-days realized variance as: $RV_{t,t+30} = \frac{365}{30} \sum_{i=1}^{30} R_{t_n}^2$ where $t = t_0 < t_1 < \cdots < t_N = T$ and $R_{t_n} = \log\left(S_{t_n}/S_{t_{n-1}}\right)$.

CAPM Beta: We calculate monthly conditional betas, using historical returns over a window of 180 days. We use data from OptionMetrics, where available, or CRSP.

Demand Pressure: We define demand pressure as the the number of contracts traded during the month at prices higher than the prevailing bid/ask quote midpoint, minus the number of contracts traded during the month at prices below the prevailing bid/ask quote midpoint, multiplied by the absolute value of the option's delta and scaled by the total trading volume across all option series.

Innovations: Innovations of a generic variable $X(t)$ are obtained using first differences, i.e., $\epsilon^X(t) = X(t) - X(t-1)$.

2 Technical Appendix

2.1 Equilibrium

For completeness, we derive all equilibrium quantities in this Appendix. However, the proofs follow grossly Basak (2005). The time-separability of the utility function and the different goods allow for such a simple form.
(i) Dynamics of the stochastic weighting process λ: Itô's Lemma applied to $\eta(t) = \xi^A(t)/\xi^B(t)$ gives:

$$d\eta(t) \;=\; \frac{d\xi^A(t)}{\xi^B(t)} - \frac{\xi^A(t)}{\left(\xi^A(t)\right)^2} d\xi^B(t) + \frac{1}{2}\frac{2\xi^A(t)}{\left(\xi^B(t)\right)^3}\left(d\xi^B(t)\right)^2 - \frac{1}{\left(\xi^B(t)\right)^2} d\xi^B(t) d\xi^A(t).$$

Since markets are complete, there exists a unique stochastic discount factor for each agent. Absence of arbitrage implies for $n = A, B$:

$$\frac{d\xi^n(t)}{\xi^n(t)} = -r(t)dt - \theta^n(D_1(t), D_2(t), z(t))' dW_Y^n,$$

where $\theta^n = (\theta^n_{D_1}(t), \theta^n_{D_2}(t), \theta^n_z(t))'$ is the vector of market prices of risk perceived by agent i. It then follows,

$$\begin{aligned} d\eta(t) \;&=\; \frac{\xi^A(t)}{\xi^B(t)}\frac{d\xi^A(t)}{\xi^A(t)} - \frac{\xi^A(t)}{\xi^B(t)}\frac{d\xi^B(t)}{\xi^B(t)} + \frac{\xi^A(t)}{\xi^B(t)}\left(\frac{d\xi^B(t)}{\xi^B(t)}\right)^2 - \frac{1}{\left(\xi^B(t)\right)^2} d\xi^B(t) d\xi^A(t), \\ &=\; \eta(t)\Big(- r(t)dt - \theta^1_A(t)dW_A(t) - \theta^1_z(t)dW_z(t) - \left(-r(t)dt - \theta^2_A(t)dW_A(t) - \theta^2_z(t)dW_z(t)\right) \\ &\quad + \Big(\left(\theta^2_A(t)\right)^2 + \left(\theta^2_z(t)\right)^2 - \theta^1_A(t)\theta^2_A(t) - \theta^1_z(t)\theta^2_z(t)\Big)\, dt\Big). \end{aligned} \tag{5}$$

The price of the stock in our economy follows the dynamics:

$$dS_i(t) = S_i(t)\left(\mu_{S_i}(t)dt + \sigma_{S_i D_1} dW_{D_1}(t) + \sigma_{S_i D_2} dW_{D_2}(t) + \sigma_{S_i z} dW_z(t)\right),$$

where $S_i(t)$ is the price of the equity of firm i, and the expected growth rates $\mu_{S_i}(t)$ and the volatility coefficients $\sigma_{S_i D_1}(t), \sigma_{S_i D_2}(t)$ and $\sigma_{S_i z}(t)$ are determined in equilibrium. It is easily shown that the difference in the perceived rates of return have to satisfy the consistency condition:

$$\mu^A_i(t) - \mu^B_i(t) = \sigma_i\left(\Psi_{D_1}(t), \Psi_{D_2}(t), \quad \alpha_{D_1}\Psi_{D_1}(t)\frac{\sigma_{D_1}}{\sigma_z} + \alpha_{D_2}\Psi_{D_2}(t)\frac{\sigma_{D_2}}{\sigma_z} + \beta\Psi_z(t)\right)',$$

where i denotes security i and $\sigma_i = (\sigma_{S_i D_1}, \sigma_{S_i D_2}, \sigma_{S_i z})$. The definition of market price of risk yields:

$$\sigma_{iD_1}\theta^n_{D_1}(t) + \sigma_{iD_2}\theta^n_{D_2}(t) + \sigma_{iz}\theta^n_z(t) = \mu^n_i(t) - r(t).$$

After some simple algebra, we obtain:

$$\begin{aligned}&\sigma_{iD_1}(t)\left(\theta^A_{D_1}(t) - \theta^B_{D_1}\right) + \sigma_{iD_2}(t)\left(\theta^A_{D_2}(t) - \theta^B_{D_2}\right) + \sigma_{nz}(t)\left(\theta^A_z(t) - \theta^B_z(t)\right) = \sigma_{iD_1}(t)\Psi_{D_1}(t) + \sigma_{iD_2}(t)\Psi_{D_2}(t) \\ &+\sigma_{nz}(t)\left(\alpha_{D_1}\Psi_{D_1}(t)\frac{\sigma_{D_1}}{\sigma_z} + \alpha_{D_2}\Psi_{D_2}(t)\frac{\sigma_{D_2}}{\sigma_z} + \beta\Psi_z(t)\right).\end{aligned}$$

Since this equation has to hold for any $\sigma_{nD_i}(t)$ and $\sigma_{nz}(t)$, it follows:

$$\begin{aligned}\theta^A_{D_i}(t) - \theta^B_{D_i}(t) &= \Psi_{D_i}(t), \\ \theta^A_z(t) - \theta^B_z(t) &= \left(\alpha_{D_1}\Psi_{D_1}(t)\frac{\sigma_{D_1}}{\sigma_z} + \alpha_{D_2}\Psi_{D_2}(t)\frac{\sigma_{D_2}}{\sigma_z} + \beta\Psi_z(t)\right).\end{aligned}$$

By construction, we also have:

$$\begin{aligned}dW_{D_i}(t) &= \frac{m^n_{D_i}(t) - \mu_{D_i}(t)}{\sigma_{D_i}}dt + dW^n_{D_i}(t), \\ dW_z(t) &= \left(\alpha_{D_1}\frac{m^n_{D_1}(t) - \mu_{D_1}(t)}{\sigma_z} + \alpha_{D_2}\frac{m^n_{D_2}(t) - \mu_{D_2}(t)}{\sigma_z} + \beta\frac{m^n_z(t) - \mu_z(t)}{\sigma_z} + dW^n_z(t)\right).\end{aligned}$$

Therefore, after substituting in equation (5), we get:

$$\begin{aligned}\frac{d\eta(t)}{\eta(t)} &= -dW^A_{D_1}(t)\Psi_{D_1}(t) - dW^A_{D_2}(t)\Psi_{D_2}(t) - \theta^A_z(t)dW^A_z(t) \\ &+\theta^B_z(t)\left(dW^A_z(t) + \alpha_{D_1}\Psi_{D_1}(t)\frac{\sigma_{D_1}}{\sigma_z} + \alpha_{D_2}\Psi_{D_2}(t)\frac{\sigma_{D_2}}{\sigma_z} + \beta\Psi_z(t)\right) \\ &+\Bigg(\left(\theta^A_{D_1}(t) - \Psi_{D_1}(t)\right)^2 + \left(\theta^A_{D_2}(t) - \Psi_{D_2}(t)\right)^2 + \theta^B_z(t)\left(\theta^B_z(t) - \theta^A_z(t)\right) \\ &-\theta^A_{D_1}(t)\left(\theta^A_{D_1}(t) - \Psi_{D_1}(t)\right) - \theta^A_{D_2}(t)\left(\theta^A_{D_2}(t) - \Psi_{D_2}(t)\right)\Bigg)dt, \\ &= -dW^A_{D_1}(t)\Psi_{D_1}(t) - dW^A_{D_2}(t)\Psi_{D_2}(t) - dW^A_z(t)\left(\alpha_{D_1}\Psi_{D_1}(t)\frac{\sigma_{D_1}}{\sigma_z} + \alpha_{D_2}\Psi_{D_2}(t)\frac{\sigma_{D_2}}{\sigma_z} + \beta\Psi_z(t)\right).\end{aligned}$$

(ii) Representative investor optimization and optimal consumption policies: The representative agent in the economy faces the following optimization problem:

$$V\left(c^A(t), c^B(t), \lambda(t)\right) = \sup_{\substack{c^A_{D_1}(t)+c^B_{D_1}(t)=D_1(t) \\ c^A_{D_2}(t)+c^B_{D_2}(t)=D_2(t)}} u^A\left(c^A_{D_1}, c^A_{D_2}\right) + \lambda(t)u^B\left(c^B_{D_1}, c^B_{D_2}\right)$$

where $\lambda(t) > 0$. Since the agents' utility functions are separable with respect to both goods, we can set up two different utility functions for the representative agent. The representative agent's utility over consumption good of tree 1 is given by:

$$U_{D_1}\left(c_{D_1}(t), \lambda(t)\right) = \sup_{c^A_{D_1}(t)+c^B_{D_1}(t)=D_1(t)} e^{-\rho t}\frac{c^A_{D_1}(t)^{1-\gamma}}{1-\gamma} + e^{-\rho t}\frac{c^B_{D_1}(t)^{1-\gamma}}{1-\gamma}$$

and similarly over good of tree 2:

$$U_{D_2}\left(c_{D_2}(t),\lambda(t)\right)=\sup_{c^A_{D_2}(t)+c^B_{D_2}(t)=D_2(t)} e^{-\rho t}\frac{c^A_{D_2}(t)^{1-\gamma}}{1-\gamma}+e^{-\rho t}\frac{c^B_{D_2}(t)^{1-\gamma}}{1-\gamma}.$$

Hence,

$$V\left(c^A(t),c^B(t),\lambda(t)\right)=U_{D_1}\left(c_{D_1},\lambda(t)\right)+U_{D_2}\left(c_{D_2},\lambda(t)\right).$$

Optimality of individual consumption plans implies that the stochastic weight takes the following form:

$$\lambda(t)=y_A\xi^A(t)/y_B\xi^B(t)=\frac{y_A}{y_B}\eta(t),$$

where $\lambda(t)$, weighted by y_A/y_B is the Radon-Nikodym derivative of investor B's beliefs with respect to investor A's beliefs. From these equations together with the market clearing conditions, we derive the solutions for the individual state price densities using the first good as a numeraire:

$$\xi^A(t)=e^{-\rho t}\frac{1}{y_A}D_1(t)^{-\gamma}\left(1+\lambda(t)^{1/\gamma}\right)^{\gamma},\quad \xi^B(t)=e^{-\rho t}\frac{1}{y_B}D_1(t)^{-\gamma}\left(1+\lambda(t)^{1/\gamma}\right)^{\gamma}\lambda(t)^{-1}.$$

Due to our separable preferences assumption, the equilibrium expressions are the same as in a one good economy. The effect of the second good appears indirectly through the stochastic weighting $\lambda(t)$ which accounts for the belief disagreement.

2.2 Belief Comovement

Lemma 1. *Let γ^n_{ij} be the steady-state conditional covariance between $m^n_i(t)$ and $m^n_j(t)$ implied by the following Riccati equation for the belief dynamics of agent $n=A,B$, where $i,j=D_1,D_2,z$.*

$$0=a_1\gamma^n+\gamma^n a_1'+b^A b'^A\gamma'^n A'\left(BB'\right)^{-1}A\gamma^n,$$

where a_1,b,A and B are defined in Section I of the main paper. It then follows, that the conditional covariance between belief disagreement is given by:

$$\begin{aligned}Cov_t(d\Psi_i,d\Psi_j) &= \Bigg[\left(\frac{\gamma^A_i-\gamma^B_i}{\sigma_i^2}\right)\left(\frac{\gamma^A_{ij}-\gamma^B_{ij}}{\sigma_i\sigma_j}\right)+\left(\frac{\gamma^A_{D_1D_2}-\gamma^B_{D_1D_2}}{\sigma_{D_1}\sigma_{D_2}}\right)\left(\frac{\gamma^A_{D_2j}-\gamma^B_{D_2j}}{\sigma_{D_2}\sigma_j}\right)\\ &+\left(\frac{\alpha_{D_1}\left(\gamma^A_{D_1}-\gamma^B_{D_1}\right)+\alpha_{D_2}\left(\gamma^A_{D_1D_2}-\gamma^B_{D_1D_2}\right)+\beta\left(\gamma^A_{D_1z}-\gamma^B_{D_1z}\right)}{\sigma_{D_1}\sigma_z}\right)\\ &\times\left(\frac{\alpha_{D_1}\left(\gamma^A_{D_1j}-\gamma^B_{D_1j}\right)+\alpha_{D_2}\left(\gamma^A_{D_2j}-\gamma^B_{D_2j}\right)+\beta\left(\gamma^A_{zj}-\gamma^B_{zj}\right)}{\sigma_j\sigma_z}\right)\Bigg],\end{aligned}$$

where for brevity we have set $\gamma^n_i:=\gamma^n_{ii}$ for $i=D_1,D_2,z$.

The expression follows directly from the dynamics of $\Psi_j(t)$ and $\Psi_i(t)$ given in Section 2.4.

2.3 Joint Laplace Transform

Lemma 2. *Under the steady state distribution, the joint Laplace transform of $D_1(t), D_2(t)$ and $\lambda(t)$ with respect to the belief of agent A is given by:*

$$E_t^A\left(\left(\frac{D_1(T)}{D_1(t)}\right)^{\epsilon_{D_1}}\left(\frac{D_2(T)}{D_2(t)}\right)^{\epsilon_{D_2}}\left(\frac{\lambda(T)}{\lambda(t)}\right)^{\chi}\right) = F_{m^A}\left(m^A, \tau; \epsilon_{D_1}, \epsilon_{D_2}\right) \times F_\Psi\left(\Psi, \tau; \epsilon_{D_1}, \epsilon_{D_2}, \chi\right), \tag{6}$$

where

$$F_{m^A}(m^A, \tau; \epsilon_{D_1}, \epsilon_{D_2}) \quad = \quad \exp\left(A_{m^A}(\tau) + B_{m^A}(\tau) m^A\right), \tag{7}$$

with $\tau = T - t$ and

$$F_\Psi\left(\Psi, \tau, \epsilon_{D_1}, \epsilon_{D_2}, \chi\right) = \exp\left(A_\Psi(\tau) + B_\Psi(\tau)\Psi + \Psi' C_\Psi(\tau)\Psi\right).$$

for functions $A_{m^A}, B_{m^A}, A_\Psi, B_\Psi$ and C_Ψ detailed in the proof.

2.4 Formulas Needed for the Proof of Lemma 2

For the sake of simplicity, in the following, we label the two trees as i and j. We first summarize the most important filtered dynamics needed to compute the Laplace transform. The disagreement dynamics of firm i are:

$$\begin{aligned} d\Psi_i(t) \quad = \quad & \left(\underbrace{\left(a_{1i} + \frac{\gamma_i^B}{\sigma_i^2}\right)}_{K_{1i}}\Psi_i(t) + \underbrace{\frac{\gamma_{ij}^B}{\sigma_j\sigma_i}}_{K_{1j}}\Psi_j(t) + \underbrace{\frac{\left(\alpha_i\gamma_i^B + \alpha_j\gamma_{ij}^B + \beta\gamma_{iz}^B\right)}{\sigma_i\sigma_z}}_{K_{1z}}\Psi_z(t)\right)dt \\ & + \underbrace{\left(\frac{\gamma_i^A - \gamma_i^B}{\sigma_i^2}\right)}_{\sigma_{1i}}dW_i^A(t) + \underbrace{\left(\frac{\gamma_{ij}^A - \gamma_{ij}^B}{\sigma_i\sigma_j}\right)}_{\sigma_{1j}}dW_j^A(t) + \underbrace{\frac{1}{\sigma_i\sigma_z}\left(\alpha_i\left(\gamma_i^A - \gamma_i^B\right) + \alpha_j\left(\gamma_{ij}^A - \gamma_{ij}^B\right) + \beta\left(\gamma_{iz}^A - \gamma_{iz}^B\right)\right)}_{\sigma_{1z}}dW_z^A(t). \end{aligned}$$

Similarly, the disagreement dynamics of firm j are given by:

$$\begin{aligned} d\Psi_j(t) \quad = \quad & \left(\underbrace{\frac{\gamma_{ij}^B}{\sigma_j\sigma_i}}_{K_{2i}}\Psi_i(t) + \underbrace{\left(a_{1j} + \frac{\gamma_j^B}{\sigma_j^2}\right)}_{K_{2j}}\Psi_j(t) + \underbrace{\frac{\alpha_i\gamma_{ij}^B + \alpha_j\gamma_j^B + \beta\gamma_{jz}^B}{\sigma_j\sigma_z}}_{K_{2z}}\Psi_z(t)\right)dt \\ & + \underbrace{\left(\frac{\gamma_{ij}^A - \gamma_{ij}^B}{\sigma_i\sigma_j}\right)}_{\sigma_{2i}}dW_i^A(t) + \underbrace{\left(\frac{\gamma_j^1 - \gamma_j^B}{\sigma_j^2}\right)}_{\sigma_{2j}}dW_j^A(t) + \underbrace{\frac{1}{\sigma_j\sigma_z}\left(\alpha_i\left(\gamma_{ij}^A - \gamma_{ij}^B\right) + \alpha_j\left(\gamma_j^A - \gamma_j^B\right) + \beta\left(\gamma_{jz}^A - \gamma_{jz}^B\right)\right)}_{\sigma_{2z}}dW_z^A(t). \end{aligned}$$

The disagreement about the signal growth rate is given by:

$$
\begin{aligned}
d\Psi_z(t) &= \left(\underbrace{\frac{\gamma_{iz}^B}{\sigma_i\sigma_z}}_{K_{3i}} \Psi_i(t) + \underbrace{\frac{\gamma_{jz}^2}{\sigma_z\sigma_j}}_{K_{3j}} \Psi_j(t) + \underbrace{\left(a_{1z} + \frac{1}{\sigma_z^2}\left(\alpha_i\gamma_{iz}^B + \alpha_j\gamma_{jz}^B + \beta\gamma_z^B\right)\right)}_{K_{3z}} \Psi_z(t) \right) dt \\
&\quad + \underbrace{\left(\frac{\gamma_{iz}^A - \gamma_{iz}^B}{\sigma_i\sigma_z} \right)}_{\sigma_{3i}} dW_i^A(t) + \underbrace{\left(\frac{\gamma_{jz}^A - \gamma_{jz}^B}{\sigma_j\sigma_z} \right)}_{\sigma_{3j}} dW_j^A(t) + \underbrace{\frac{1}{\sigma_z^2}\left(\alpha_i\left(\gamma_{iz}^A - \gamma_{iz}^B\right) + \alpha_j\left(\gamma_{jz}^A - \gamma_{jz}^B\right) + \beta\left(\gamma_z^A - \gamma_z^B\right)\right)}_{\sigma_{3z}} dW_z^A(t).
\end{aligned}
$$

The growth rate of firm i is given by:

$$
dm_i^A(t) = \left(a_{0i} + a_{1i}m_i^A(t)\right)dt + \underbrace{\frac{\gamma_i^A}{\sigma_i}}_{\sigma_{4i}} dW_i^A(t) + \underbrace{\frac{\gamma_{ij}^A}{\sigma_j}}_{\sigma_{4j}} dW_j^A(t) + \underbrace{\frac{\alpha_i\gamma_i^A + \alpha_i\gamma_{ij}^A + \beta\gamma_{iz}^A}{\sigma_z}}_{\sigma_{4z}} dW_z^A(t).
$$

Similarly, the growth rate of firm j is given by:

$$
dm_j^A(t) = \left(a_{0j} + a_{1j}m_j^A(t)\right)dt + \underbrace{\frac{\gamma_{ij}^A}{\sigma_i}}_{\sigma_{5i}} dW_i^A(t) + \underbrace{\frac{\gamma_j^A}{\sigma_j}}_{\sigma_{5j}} dW_j^A(t) + \underbrace{\frac{\alpha_i\gamma_{ij}^A + \alpha_j\gamma_j^A + \beta\gamma_{jz}^A}{\sigma_z}}_{\sigma_{5z}} dW_z^A(t).
$$

The Radon-Nikodym derivative in this economy evolves according to:

$$
\frac{d\eta(t)}{\eta(t)} = -\left[\Psi_i(t)dW_i^A + \Psi_j(t)dW_j^A + \left(\underbrace{\frac{\alpha_i\sigma_i}{\sigma_z}}_{\bar{\alpha}_i} \Psi_i(t) + \underbrace{\frac{\alpha_j\sigma_j}{\sigma_z}}_{\bar{\alpha}_j} \Psi_j(t) + \beta\Psi_z(t) \right) dW_z^A(t) \right].
$$

Finally, the fundamentals have the following dynamics:

$$
\begin{aligned}
dD_i(t)/D_i(t) &= m_i^A(t)dt + \underbrace{\sigma_i}_{\sigma_{7i}} dW_i(t)^A \\
dD_j(t)/D_j(t) &= m_j^A(t)dt + \underbrace{\sigma_j}_{\sigma_{7j}} dW_j(t)^A
\end{aligned}
$$

Let me collect all variables in the vector $X = (\Psi_i, \Psi_j, \Psi_z, m_i, m_j, \eta, D_1, D_2)'$, and all shocks in the vector $W^A = \left(W_i^A, W_j^A, W_z^A\right)'$. We can now compactly write the dynamics of X as:

$$dX = \left[\underbrace{\begin{pmatrix} 0 \\ 0 \\ 0 \\ a_{0i} \\ a_{0j} \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{K_0} + \underbrace{\begin{pmatrix} K_{1i} & K_{1j} & K_{1z} & 0 & 0 & 0 & 0 & 0 \\ K_{2i} & K_{2j} & K_{2z} & 0 & 0 & 0 & 0 & 0 \\ K_{3i} & K_{3j} & K_{3z} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{1i} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{1j} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & D_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & D_2 & 0 & 0 \end{pmatrix}}_{K_1} X(t)\right] dt + \underbrace{\begin{pmatrix} \sigma_{1i} & \sigma_{1j} & \sigma_{1z} \\ \sigma_{2i} & \sigma_{2j} & \sigma_{2z} \\ \sigma_{3i} & \sigma_{3j} & \sigma_{3z} \\ \sigma_{4i} & \sigma_{4j} & \sigma_{4z} \\ \sigma_{5i} & \sigma_{5j} & \sigma_{5z} \\ -\eta\hat{\sigma}_{6i} & -\eta\hat{\sigma}_{6j} & -\eta\hat{\sigma}_{6z} \\ D_1\sigma_{7i} & 0 & 0 \\ 0 & D_2\sigma_{8j} & 0 \end{pmatrix}}_{S} dW^A(t),$$

where hats in $\hat{\sigma}_{6i}, \hat{\sigma}_{6j}$ and $\hat{\sigma}_{6z}$ shall remind us that diffusion of $\frac{d\eta}{\eta}$ is a function of $\Psi(t)$. Let me also denote those sub-blocks (yellow ones) of K_0, K_1, S, which are related to $\Psi(t)$ by:

$$\begin{aligned} \kappa_0 &:= K_{0(1:3)} = 0_{3\times 1} \\ \kappa_1 &:= K_{1(1:3,1:3)} \\ \Sigma &:= S_{(1:3,1:3)}. \end{aligned}$$

Thus, the dynamics of $\Psi(t)$ can be compactly written as:

$$d\Psi(t) = (\kappa_0 + \kappa_1 \Psi(t))\, dt + \Sigma dW^A(t).$$

Let $m^A(t) = (m_{D_1}^A(t) m_{D_2}^A(t) m_z^A(t))$, then the joint Laplace transform of $X(t)$ is given by:

$$F_X = D_1^{\varepsilon_{D_1}} D_2^{\varepsilon_{D_2}} \eta^{\chi} F_{m^A} F_\Psi, \tag{8}$$

where F_{m^A} is affine in m_i and m_j:

$$F_{m^A} = \exp\left(A_{m^A} + \bar{A}_i(\tau) m_i + \bar{A}_j(\tau) m_j\right),$$

and F_{Ψ_i,Ψ_j,Ψ_z} is affine-quadratic in $\Psi = (\Psi_i, \Psi_j, \Psi_z)$:

$$F_{\Psi_i,\Psi_j,\Psi_z} = \exp(A(\tau) + B(\tau)'\Psi + \Psi' C(\tau)\Psi).$$

In matrix notation, Feynman-Kač implies:

$$\frac{\partial F}{\partial t} + \frac{\partial F}{\partial X'}(K_0 + K_1 X) + \frac{1}{2} Tr\left(\frac{\partial^2 F}{\partial X \partial X'} SS'\right) = 0$$

or

$$\frac{\partial F}{\partial t} + \sum_{i=1}^{11} \frac{\partial F}{\partial X_i}(K_0(i) + K_1(i,:)X) + \frac{1}{2}\sum_{i,j=1}^{11} C_{ij} \frac{\partial^2 F}{\partial X_i \partial X_j} = 0,$$

where C_{ij} is the $ij-$th element of the matrix $C = SS'$. The goal is to reduce the problem of solving the PDE in F_X, to solving simply the PDE in F_Ψ, which can be done virtually in closed form. This requires several adjustments to the parameters of Ψ dynamics and the constants in the respective ODEs. More specifically, $A(\tau), B(\tau), C(\tau)$ solve the following system:

$$
\begin{aligned}
\frac{\partial A}{\partial \tau} &= B(\tau)'\bar{\kappa}_0 + Tr\left[C(\tau)\Sigma\Sigma'\right] + \frac{1}{2}B(\tau)'\Sigma\Sigma' B(\tau) + A_c \qquad \text{(ODE.1)} \\
\frac{\partial B'}{\partial \tau}\Psi &= B(\tau)'\bar{\kappa}_1\Psi + 2\bar{\kappa}_0' C(\tau)\Psi + 2Tr\left[B'(\tau)\Sigma\Sigma' C(\tau)\Psi\right] + B_c'\Psi \qquad \text{(ODE.2)} \\
Tr\left[\frac{\partial C'}{\partial \tau}\Psi\Psi'\right] &= Tr\left[\left(C(\tau)\bar{\kappa}_1 + \bar{\kappa}_1' C(\tau)\right)\Psi\Psi'\right] + 2Tr\left[C(\tau)\Sigma\Sigma' C(\tau)\Psi\Psi'\right] + Tr\left(C_c\Psi\Psi'\right) \qquad \text{(ODE.3)},
\end{aligned}
\tag{9}
$$

where $\bar{\kappa}_0, \bar{\kappa}_1$ denote appropriately adjusted coefficients κ_0, κ_1, and A_c, B_c, C_c are the adjusted constants.

Since the adjustments involve the cross-derivatives of F with respect to X, we spell out the Hessian matrix for completeness:

$$
\frac{\partial^2}{\partial X \partial X'} = \begin{pmatrix}
\frac{\partial^2}{\partial \Psi_i^2} & \frac{\partial^2}{\partial \Psi_i \partial \Psi_j} & \frac{\partial^2}{\partial \Psi_i \partial \Psi_z} & \frac{\partial^2}{\partial \Psi_i \partial m_{D_2}} & \frac{\partial^2}{\partial \Psi_i \partial m_{D_2}} & \frac{\partial^2}{\partial \Psi_i \partial \eta} & \frac{\partial^2}{\partial \Psi_i \partial D_1} & \frac{\partial^2}{\partial \Psi_i \partial D_2} \\
 & \frac{\partial^2}{\partial \Psi_j^2} & \frac{\partial^2}{\partial \Psi_j \partial \Psi_z} & \frac{\partial^2}{\partial \Psi_j \partial m_{D_2}} & \frac{\partial^2}{\partial \Psi_j \partial m_{D_2}} & \frac{\partial^2}{\partial \Psi_{D_2} \partial \eta} & \frac{\partial^2}{\partial \Psi_j \partial D_1} & \frac{\partial^2}{\partial \Psi_j \partial D_2} \\
 & & \frac{\partial^2}{\partial \Psi_z^2} & \frac{\partial^2}{\partial \Psi_z \partial m_{D_1}} & \frac{\partial^2}{\partial \Psi_z \partial m_{D_2}} & \frac{\partial^2}{\partial \Psi_z \partial \eta} & \frac{\partial^2}{\partial \Psi_z \partial D_1} & \frac{\partial^2}{\partial \Psi_z \partial D_2} \\
 & & & \frac{\partial^2}{\partial m_{D_2}^2} & \frac{\partial^2}{\partial m_{D_2} \partial m_{D_2}} & \frac{\partial^2}{\partial m_{D_2} \partial \eta} & \frac{\partial^2}{\partial m_{D_2} \partial D_1} & \frac{\partial^2}{\partial m_{D_2} \partial D_2} \\
 & & & & \frac{\partial^2}{\partial m_{D_2}^2} & \frac{\partial^2}{\partial m_{D_2} \partial \eta} & \frac{\partial^2}{\partial m_{D_2} \partial D_1} & \frac{\partial^2}{\partial m_{D_2} \partial D_2} \\
 & & & & & \frac{\partial^2}{\partial \eta^2} & \frac{\partial^2}{\partial \eta \partial D_1} & \frac{\partial^2}{\partial \eta \partial D_2} \\
 & & & & & & \frac{\partial^2}{\partial D_1^2} & \frac{\partial^2}{\partial D_1 \partial D_2} \\
 & & & & & & & \frac{\partial^2}{\partial D_2^2}
\end{pmatrix}
$$

Types of adjustments:

1. Terms involving derivatives of the type $\frac{\partial^2}{\partial \Psi_i \partial m_j}, i = \{D_1, D_2, z\}, j = \{D_1, D_2\}$ adjust the vector κ_0 in the drift of Ψ.
2. Terms involving derivatives of the type $\frac{\partial^2}{\partial \Psi_i \partial D_1}, \frac{\partial^2}{\partial \Psi_i \partial D_2}, i = \{D_1, D_2, z\}$ adjust the vector κ_0 in the drift of Ψ.
3. Terms involving derivatives of the type $\frac{\partial^2}{\partial \Psi_i \partial \eta}, i = \{D_1, D_2, z\}$ adjust the mean reversion matrix κ_1 in the drift of Ψ.
4. Terms involving derivatives of the type $\frac{\partial^2}{\partial m_i \partial \eta}, i = \{D_1, D_2\}$ adjust the constant vector B_c in the ODE.2.
5. Terms involving derivatives of the type $\frac{\partial^2}{\partial \eta \partial D_1}, \frac{\partial^2}{\partial \eta \partial D_2}$ adjust the constant vector B_c in the ODE.2.
6. Terms involving derivatives of the type $\frac{\partial^2}{\partial \eta^2}$ adjust the constant matrix C_c in the ODE.3.

For all the cases above, we need to consider the structure of $\frac{1}{dt}\frac{\partial^2}{\partial X_i \partial X_j}\langle dX_i, dX_j\rangle$. Below we neglect the division by $\frac{1}{dt}$ for brevity. Let us start with the adjustments of the constant drift matrix κ_0.

Adjustment 1: $\frac{\partial^2 F}{\partial \Psi_i \partial m_j}$ where $i = \{D_1, D_2, z\}$ and $j = \{D_1, D_2\}$

$$
\begin{array}{ccc}
\frac{\partial^2 F}{\partial \Psi_i \partial m_{D_2}} = \frac{\partial F}{\partial \Psi_i}\bar{A}_{D_1} & \text{and} & \langle d\Psi_i, dm_{D_2}\rangle = \sum_i \sigma_{1i}\sigma_{5i} \\
\frac{\partial^2 F}{\partial \Psi_i \partial m_{D_2}} = \frac{\partial F}{\partial \Psi_i}\bar{A}_{D_2} & \text{and} & \langle d\Psi_i, dm_{D_2}\rangle = \sum_i \sigma_{1i}\sigma_{6i} \\
... & \text{and} & ...
\end{array}
$$

We need to add the following vector to κ_0 :

$$\kappa_0^{adj1} = \begin{pmatrix} \bar{A}_{D_1}\sum_i \sigma_{1i}\sigma_{5i} + \bar{A}_{D_2}\sum_i \sigma_{1i} \\ \bar{A}_{D_1}\sum_i \sigma_{2i}\sigma_{5i} + \bar{A}_{D_2}\sum_i \sigma_{2i} \\ \bar{A}_{D_1}\sum_i \sigma_{3i}\sigma_{5i} + \bar{A}_{D_2}\sum_i \sigma_{3i} \end{pmatrix} = \underbrace{\begin{pmatrix} \sigma_{(1,:)} \\ \sigma_{(2,:)} \\ \sigma_{(3,:)} \end{pmatrix}}_{3\times 3} \underbrace{\left(\bar{A}_{D_1}\sigma'_{(5,:)} + \bar{A}_{D_2}\sigma'_{(6,:)}\right)}_{3\times 1}.$$

Adjustment 2: $\frac{\partial^2 F}{\partial \Psi_i \partial I}, i = \{D_1, D_2, z\}, I = \{D_1, D_2\}$

$$\begin{array}{rclcc} \frac{\partial^2 F}{\partial \Psi_i \partial D_1} & = & \frac{\partial F}{\partial \Psi_i}\frac{\varepsilon_{D_1}}{D_1} & \text{and} & \langle d\Psi_i, dD_1\rangle = D_1\sigma_{1i}\sigma_{9D_1} \\ \frac{\partial^2 F}{\partial \Psi_i \partial D_2} & = & \frac{\partial F}{\partial \Psi_i}\frac{\varepsilon_{D_2}}{D_2} & \text{and} & \langle d\Psi_i, dD_1\rangle = D_2\sigma_{1j}\sigma_{10D_2} \\ & ... & & \text{and} & ... \end{array}$$

Adjustment 2 adds the following vector to κ_0:

$$\kappa_0^{adj2} = \underbrace{\begin{pmatrix} \varepsilon_{D_1}\sigma_{1i}\sigma_{9D_1} + \varepsilon_{D_2}\sigma_{1j}\sigma_{10D_2} \\ \varepsilon_{D_1}\sigma_{2D_1}\sigma_{9D_1} + \varepsilon_{D_2}\sigma_{2D_2}\sigma_{10D_2} \\ 0 \end{pmatrix}}_{3\times 1},$$

where 0 in the last row of κ_0^{adj2} follows from the independence between the disagreement on the signal and D_1 and D_2 dynamics.

Summarizing, Adj 1 and **Adj 2** imply that the adjusted vector $\bar{\kappa}_0$ in the drift of Ψ is:

$$\boxed{\bar{\kappa}_0 = \kappa_0 + \kappa_0^{adj1} + \kappa_0^{adj2}.}$$

Adjustment 3 Next consider the terms that can be absorbed into the mean reversion matrix κ_1. Those involve $\frac{\partial^2 F}{\partial \Psi_i \partial \eta}, i = \{D_1, D_2, z\}$.

$$\begin{array}{rclcc} \frac{\partial^2 F}{\partial \Psi_i \partial \eta} & = & \frac{\partial F}{\partial \Psi_i}\frac{\chi}{\eta} & \text{and} & \langle d\Psi_{D_1}, d\eta\rangle = -\eta\left[\left(\sigma_{1i} + \sigma_{1z}\bar{\alpha}_{D_1}\right)\Psi_i + \left(\sigma_{1j} + \sigma_{1z}\bar{\alpha}_{D_2}\right)\Psi_j + \sigma_{1z}\beta\Psi_z\right] \\ & & & \Rightarrow & \kappa_{1(1,:)}^{adj} = -\chi\left(\begin{array}{ccc} \left(\sigma_{1i} + \sigma_{1z}\bar{\alpha}_{D_1}\right), & \left(\sigma_{1j} + \sigma_{1z}\bar{\alpha}_{D_2}\right), & \sigma_{1z}\beta \end{array}\right)_{1\times 3} \\ \frac{\partial^2 F}{\partial \Psi_j \partial \eta} & = & \frac{\partial F}{\partial \Psi_j}\frac{\chi}{\eta} & \text{and} & \langle d\Psi_{D_2}, d\eta\rangle = -\eta\left[\left(\sigma_{2D_1} + \sigma_{2z}\bar{\alpha}_{D_1}\right)\Psi_i + \left(\sigma_{2D_2} + \sigma_{2z}\bar{\alpha}_{D_2}\right)\Psi_j + \sigma_{2z}\beta\Psi_z\right] \\ & & & \Rightarrow & \kappa_{1(2,:)}^{adj} = -\chi\left(\begin{array}{ccc} \left(\sigma_{2D_1} + \sigma_{2z}\bar{\alpha}_{D_1}\right), & \left(\sigma_{2D_2} + \sigma_{2z}\bar{\alpha}_{D_2}\right), & \sigma_{2z}\beta \end{array}\right)_{1\times 3} \\ \frac{\partial^2 F}{\partial \Psi_z \partial \eta} & = & \frac{\partial F}{\partial \Psi_z}\frac{\chi}{\eta} & \text{and} & \langle d\Psi_z, d\eta\rangle = -\eta\left[\left(\sigma_{4D_1} + \sigma_{4z}\bar{\alpha}_{D_1}\right)\Psi_i + \left(\sigma_{4D_2} + \sigma_{4z}\bar{\alpha}_{D_2}\right)\Psi_j + \sigma_{4z}\beta\Psi_z\right] \\ & & & \Rightarrow & \kappa_{1(3,:)}^{adj} = -\chi\left(\begin{array}{ccc} \left(\sigma_{4D_1} + \sigma_{4z}\bar{\alpha}_{D_1}\right), & \left(\sigma_{4D_2} + \sigma_{4z}\bar{\alpha}_{D_2}\right), & \sigma_{4z}\beta \end{array}\right)_{1\times 3} \end{array}$$

Thus,

$$\kappa_1^{adj} = -\chi \underbrace{\begin{bmatrix} \left(\sigma_{1i} + \sigma_{1z}\bar{\alpha}_{D_1}\right) & \left(\sigma_{1D_2} + \sigma_{1z}\bar{\alpha}_{D_2}\right) & \sigma_{1z}\beta \\ \left(\sigma_{2D_1} + \sigma_{2z}\bar{\alpha}_{D_1}\right) & \left(\sigma_{2D_2} + \sigma_{2z}\bar{\alpha}_{D_2}\right) & \sigma_{2z}\beta \\ \left(\sigma_{3D_1} + \sigma_{3z}\bar{\alpha}_{D_1}\right) & \left(\sigma_{3D_2} + \sigma_{3z}\bar{\alpha}_{D_2}\right) & \sigma_{3z}\beta \end{bmatrix}}_{3\times 3}$$

Summarizing, the adjusted mean reversion matrix in the dynamics of Ψ is given by:

$$\boxed{\bar{\kappa}_1 = \kappa_1 + \kappa_1^{adj}.}$$

Adjustment 4: $\frac{\partial^2 F}{\partial m_i \partial \eta}, i = \{D_1, D_2\}$

$$\begin{array}{lll} \frac{\partial^2 F}{\partial m_{D_2} \partial \eta} = \frac{\chi \bar{A}_{D_1}}{\eta} F & \text{and} & \langle dm_{D_2}, d\eta \rangle = -\eta \left[\left(\sigma_{4i} + \sigma_{5z}\bar{\alpha}_{D_1}\right) \Psi_i + \left(\sigma_{5D_2} + \sigma_{5z}\bar{\alpha}_{D_2}\right) \Psi_j + \left(\sigma_{5z}\beta\right) \Psi_z \right] \\ \frac{\partial^2 F}{\partial m_{D_2} \partial \eta} = \frac{\chi \bar{A}_{D_2}}{\eta} F & \text{and} & \langle dm_{D_2}, d\eta \rangle = -\eta \left[\left(\sigma_{6D_1} + \sigma_{6z}\bar{\alpha}_{D_1}\right) \Psi_i + \left(\sigma_{6D_2} + \sigma_{6z}\bar{\alpha}_{D_2}\right) \Psi_j + \left(\sigma_{6z}\beta\right) \Psi_z \right] \end{array}$$

Since these equations do not involve any derivatives of the type $\frac{\partial}{\partial \Psi_i}$, but do involve linear terms in Ψ_i, we realize that they adjust the constant B_c in ODE.2:

$$B_c^{adj1} = -\chi \begin{pmatrix} \left[\bar{A}_{D_1}\left(\sigma_{4i} + \sigma_{5z}\bar{\alpha}_{D_1}\right) + \bar{A}_{D_2}\left(\sigma_{6D_1} + \sigma_{6z}\bar{\alpha}_{D_1}\right)\right] \\ \left[\bar{A}_{D_1}\left(\sigma_{5D_2} + \sigma_{5z}\bar{\alpha}_{D_2}\right) + \bar{A}_{D_2}\left(\sigma_{6D_2} + \sigma_{6z}\bar{\alpha}_{D_2}\right)\right] \\ \left[\bar{A}_{D_1}\left(\sigma_{5z}\beta\right) + \bar{A}_{D_2}\left(\sigma_{6z}\beta\right)\right] \end{pmatrix}$$

Adjustment 5: The same logic applies to the derivatives of the form: $\frac{\partial^2 F}{\partial \eta \partial I}, I = \{D_1, D_2\}$

$$\begin{array}{lll} \frac{\partial^2 F}{\partial \eta \partial D_1} = \frac{\chi \varepsilon_{D_1}}{\eta D_1} F & \text{and} & \langle d\eta, dD_1 \rangle = -\eta D_1 \sigma_{9D_1} \Psi_i \\ \frac{\partial^2 F}{\partial \eta \partial D_2} = \frac{\chi \varepsilon_{D_2}}{\eta D_2} F & \text{and} & \langle d\eta, dD_2 \rangle = -\eta D_2 \sigma_{10D_2} \Psi_j \end{array}$$

Thus the second adjustment to B_c is:

$$B_c^{adj2} = -\chi \begin{pmatrix} \varepsilon_{D_1}\sigma_{9D_1} \\ \varepsilon_{D_2}\sigma_{10D_2} \\ 0 \end{pmatrix}$$

Summarizing, the constant vector in ODE.2 is given by:

$$\boxed{B_c = B_c^{adj1} + B_c^{adj2}}$$

Adjustment 6:

$$\frac{1}{2}\frac{\partial^2 F}{\partial \eta^2} = \frac{1}{2}\frac{\chi(\chi-1)F}{\eta^2}$$

$$\langle d\eta, d\eta \rangle = \eta^2 \left(\hat{\sigma}_{8D_1}^2 + \hat{\sigma}_{8D_2}^2 + \hat{\sigma}_{8z}^2\right) = Tr \left[\begin{pmatrix} (1+\bar{\alpha}_{D_1}^2) & \bar{\alpha}_{D_1}\bar{\alpha}_{D_2} & \bar{\alpha}_{D_1}\beta \\ \bar{\alpha}_{D_1}\bar{\alpha}_{D_2} & (1+\bar{\alpha}_{D_2}^2) & \bar{\alpha}_{D_2}\beta \\ \bar{\alpha}_{D_1}\beta & \bar{\alpha}_{D_2}\beta & \beta^2 \end{pmatrix} \Psi\Psi' \right]$$

Thus, the **adjustment** of the constant matrix in ODE.3 is given by:

$$\boxed{C_c = \frac{1}{2}\chi(\chi-1) \begin{pmatrix} (1+\bar{\alpha}_{D_1}^2) & \bar{\alpha}_{D_1}\bar{\alpha}_{D_2} & \bar{\alpha}_{D_1}\beta \\ \bar{\alpha}_{D_1}\bar{\alpha}_{D_2} & (1+\bar{\alpha}_{D_2}^2) & \bar{\alpha}_{D_2}\beta \\ \bar{\alpha}_{D_1}\beta & \bar{\alpha}_{D_2}\beta & \beta^2 \end{pmatrix}}$$

We can now obtain the coefficients $A(\tau), B(\tau), C(\tau)$ by solving the system:

$$\frac{\partial A}{\partial \tau} = B(\tau)'\bar{\kappa}_0 + Tr\left[C(\tau)\Sigma\Sigma'\right] + \frac{1}{2}B'(\tau)\Sigma\Sigma' B(\tau) + A_c \qquad \text{(ODE.1)} \tag{10}$$

$$\frac{\partial B'}{\partial \tau} = B(\tau)'\bar{\kappa}_1 + 2\bar{\kappa}_0' C(\tau) + 2B'(\tau)\Sigma\Sigma' C(\tau) + B_c' \qquad \text{(ODE.2)} \tag{11}$$

$$\frac{\partial C}{\partial \tau} = C(\tau)\bar{\kappa}_1 + \bar{\kappa}_1' C(\tau) + 2C(\tau)\Sigma\Sigma' C(\tau) + C_c \qquad \text{(ODE.3)},$$

where $A(0) = 0, B(0) = 0, C(0) = 0$.

Last equation (ODE.3) can be solved in closed form by matrix Riccati linearization:

$$C(\tau) = F_{22}(\tau)^{-1} F_{21}(\tau),$$

where

$$\begin{pmatrix} F_{11}(\tau) & F_{12}(\tau) \\ F_{21}(\tau) & F_{22}(\tau) \end{pmatrix} = \exp\left[\tau \begin{pmatrix} \bar{\kappa}_1 & -2\Sigma\Sigma' \\ C_c & -\bar{\kappa}_1 \end{pmatrix}\right].$$

Given $C(\tau)$, We can obtain the solution for ODE.2 in closed form. First, for tractability, We re-write ODE.2 as:

$$\frac{\partial B}{\partial \tau} = \widetilde{\kappa}_1 B(\tau) + \widetilde{\kappa}_0,$$

where

$$\widetilde{\kappa}_1 = \bar{\kappa}_1 + 2C(\tau)\Sigma\Sigma'$$
$$\widetilde{\kappa}_0 = 2C(\tau)\bar{\kappa}_0 + B_c.$$

The solution for $B(\tau)$ is follows as (see e.g. Laub, 2005):

$$B(\tau) = \int_0^\tau e^{\widetilde{\kappa}_1(\tau - s)} \widetilde{\kappa}_0 ds.$$

This matrix integral admits an explicit solution (see Thm.1 in van Loan, 1978). Defining the following auxiliary matrix

$$C_{aux} = \begin{pmatrix} \widetilde{\kappa}_1 & \widetilde{\kappa}_0 \\ 0_{3\times 1} & 0_{1\times 1} \end{pmatrix},$$

$B(\tau)$ is represented as the upper right subblock of the following matrix exponential

$$\exp(\tau C_{aux}) = \begin{pmatrix} F_1(\tau) & B(\tau) \\ 0 & F_2(\tau) \end{pmatrix}.$$

This is the correct solution if $\widetilde{\kappa}_0, \widetilde{\kappa}_1$ are not time dependent. However, alternatively, the system can be solved numerically (and efficiently) using, e.g., the Runge-Kutta method.

We can proceed in an analogous way to obtain the coefficients in

$$F_m = e^{\bar{C}(\tau) + \bar{A}_{D_1}(\tau) m_{D_2} + \bar{A}_{D_2}(\tau) m_{D_2}} = e^{\bar{C}(\tau) + \bar{A}(\tau)' m}$$

where $m = (m_{D_2}, m_{D_2})'$. Let $J = (D_1, D_2)', \bar{W} = (W_{D_1}, W_{D_2})$.

$$dm = \left[\underbrace{\begin{pmatrix} a_{0i} \\ a_{0j} \end{pmatrix}}_{l_0} + \underbrace{\begin{pmatrix} a_{1i} & 0 \\ 0 & a_{1j} \end{pmatrix}}_{l_1} m\right] dt + \underbrace{\begin{pmatrix} \sigma_{4i} & \sigma_{5j} \\ \sigma_{6i} & \sigma_{6j} \end{pmatrix}}_{L} d\bar{W}$$

$$dJ = \left[\underbrace{\begin{pmatrix} 0 \\ 0 \end{pmatrix}}_{\lambda_0} + \underbrace{\begin{pmatrix} D_1 & 0 \\ 0 & D_2 \end{pmatrix}}_{\lambda_1} J\right] dt + \underbrace{\begin{pmatrix} D_1\sigma_{9D_1} & 0 \\ 0 & D_2\sigma_{10D_2} \end{pmatrix}}_{\Lambda} d\bar{W}$$

Our goal is to reduce the problem of solving PDE in F_X, to solving just the PDE in F_m, which can be done virtually in closed form:

$$\frac{\partial F_m}{\partial t} + \frac{\partial F_m}{\partial m'} \underbrace{(l_0 + l_1 m)}_{\mu_m} + \frac{1}{2} Tr \left[\frac{\partial^2 F_m}{\partial m \partial m'} LL' \right] = 0.$$

Which can be solved in two ODEs:

$$\frac{\partial \bar{A}'}{\partial \tau} m = \bar{A}(\tau)' \bar{l}_1 m + \bar{A}_c m \qquad \text{(ODE.1.1)}$$

$$\frac{\partial \bar{C}}{\partial \tau} = \bar{A}(\tau)' \bar{l}_0 + \frac{1}{2} \bar{A}(\tau) LL' \bar{A}(\tau) + \bar{C}_c \qquad \text{(ODE.1.2)}$$

To this end, we apply the following adjustments:

1. Terms involving derivatives of the type $\frac{\partial}{\partial J}, J = \{D_1, D_2\}$ adjust the constant vector in ODE.1.1.
2. Terms involving derivatives of the type $\frac{\partial^2}{\partial m_i \partial I}, i = \{D_1, D_2\}, I = \{D_1, D_2\}$ adjust the vector l_0 in the drift of m.
3. Terms involving derivatives of the type $\frac{\partial^2}{\partial J_i \partial J_j}, J = \{D_1, D_2\}$ adjust the constant in ODE.1.2.

Adjustment 1: $\frac{\partial F}{\partial J}, J = \{D_1, D_2\}$

$$\begin{array}{lll} \frac{\partial F}{\partial D_1} = \frac{\varepsilon_{D_1}}{D_1} F & \text{and} & \mu_{D_1} = D_1 m_{D_2} \\ \frac{\partial F}{\partial D_2} = \frac{\varepsilon_{D_2}}{D_2} F & \text{and} & \mu_{D_2} = D_2 m_{D_2} \end{array}$$

Adjustment 1 adds the following vector to the constant $\bar{A}_c$ in ODE.1.1:

$$\bar{A}_c^{adj1} = \begin{pmatrix} \varepsilon_{D_1} \\ \varepsilon_{D_2} \end{pmatrix}.$$

$$\bar{A}_c = \bar{A}_c^{adj1}.$$

Adjustment 2: $\frac{\partial^2 F}{\partial m_i \partial J_j}$

$$\begin{array}{lll} \frac{\partial^2 F}{\partial m_{D_2} \partial D_1} = \frac{\partial F}{\partial m_{D_2}} \frac{\varepsilon_{D_1}}{D_1} = \bar{A}_{D_1} \frac{\varepsilon_{D_1}}{D_1} F & \text{and} & \langle dm_{D_2}, dD_1 \rangle = D_1 \sigma_{9D_1} \sigma_{4i} \\ \frac{\partial^2 F}{\partial m_{D_2} \partial D_2} = \frac{\partial F}{\partial m_{D_2}} \frac{\varepsilon_{D_2}}{D_2} = \bar{A}_{D_1} \frac{\varepsilon_{D_2}}{D_2} F & \text{and} & \langle dm_{D_2}, dD_2 \rangle = D_1 \sigma_{10D_2} \sigma_{4i} \\ \ldots & & \ldots \end{array}$$

Adjustment 2 adds the following vector to l_0

$$l_0^{adj1} = \begin{pmatrix} \varepsilon_{D_1} \sigma_{9D_1} \sigma_{4i} + \varepsilon_{D_2} \sigma_{10D_2} \sigma_{5D_2} \\ \varepsilon_{D_1} \sigma_{9D_1} \sigma_{6D_1} + \varepsilon_{D_2} \sigma_{10D_2} \sigma_{6D_2} \\ \varepsilon_{D_1} \sigma_{9D_1} \sigma_{7D_1} + \varepsilon_{D_2} \sigma_{10D_2} \sigma_{7D_2} \end{pmatrix}$$

$$= \begin{pmatrix} \gamma_i^A \varepsilon_{D_1} + \gamma_{ij}^A \varepsilon_{D_2} \\ \gamma_{ij}^A \varepsilon_{D_1} + \gamma_{D_2}^1 \varepsilon_{D_2} \end{pmatrix}$$

$$\bar{l}_0 = l_0 + l_0^{adj1}.$$

Adjustment 3 $\frac{\partial^2 F}{\partial J_i \partial J_j}, J = \{D_1, D_2\}$, i.e.$\frac{1}{2}\sum_{ij}\frac{\partial^2 F}{\partial J_i \partial J_j}C_{ij} = \frac{1}{2}Tr\left(\frac{\partial^2 F}{\partial J \partial J'}\Lambda\Lambda'\right)$

$$\begin{array}{lcl} \frac{\partial^2 F}{\partial D_1^2} = \frac{\varepsilon_{D_1}(\varepsilon_{D_1}-1)}{D_1^2} & \text{and} & \langle dD_1\rangle = D_1^2\sigma_{9D_1}^2 \\ \frac{\partial^2 F}{\partial D_1 \partial D_2} = \frac{\varepsilon_{D_1}\varepsilon_{D_2}}{D_1 D_2} & \text{and} & \langle dD_1, dD_2\rangle = 0 \\ \ldots & & \ldots \end{array}$$

$$\begin{aligned} \bar{C}_c &= \frac{1}{2}Tr\left[\begin{pmatrix} \varepsilon_{D_1}(\varepsilon_{D_1}-1) & \varepsilon_{D_1}\varepsilon_{D_2} \\ \varepsilon_{D_1}\varepsilon_{D_2} & \varepsilon_{D_2}(\varepsilon_{D_2}-1) \end{pmatrix}\begin{pmatrix} \sigma_{9D_1}^2 & 0 \\ 0 & \sigma_{10D_2}^2 \end{pmatrix}\right] \\ &= \frac{1}{2}\left[\varepsilon_{D_1}(\varepsilon_{D_1}-1)\sigma_{9D_1}^2 + \varepsilon_{D_2}(\varepsilon_{D_2}-1)\sigma_{10D_2}^2\right] \end{aligned}$$

The system of ODEs (1.1–1.2) can now be solved explicitly:

$$\frac{\partial \bar{A}'}{\partial \tau} = \bar{A}(\tau)'\bar{l}_1 + \bar{A}_c \qquad \text{(ODE.1.1)}$$

$$\frac{\partial \bar{C}}{\partial \tau} = \bar{A}(\tau)'\bar{l}_0 + \frac{1}{2}\bar{A}(\tau)LL'\bar{A}(\tau) + \bar{C}_c \qquad \text{(ODE.1.2)}$$

It is easy to see that since l_1 is a diagonal matrix, the equations in ODE.1.1 are uncoupled, i.e. can be solved explicitly element-by-element.

$$\begin{aligned} \frac{\partial \bar{A}_i}{\partial \tau} &= \bar{A}_i(\tau)l_{1,ii} + \bar{A}_{c,i}, \qquad \bar{A}_i(0) = 0 \\ \bar{A}_i(\tau) &= -\frac{\bar{A}_{c,i}}{l_{1,ii}}\left(1 - e^{l_{1,ii}\tau}\right). \end{aligned}$$

Given this solution ODE.1.2. is obtained by direct integration.

2.5 Pricing

Lemma 3. *Let*

$$G(t,T,x_{D_1},x_{D_2};\Psi) \equiv \int_0^\infty \left(\frac{1+\lambda(T)^{1/\gamma}}{1+\lambda(t)^{1/\gamma}}\right)^\gamma \left[\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left(\frac{\lambda(T)}{\lambda(t)}\right)^{-i\chi} F_\Psi\left(\Psi,t,T;x,i\chi\right)d\chi\right]\frac{d\lambda(T)}{\lambda(T)},$$

where functions F_Ψ and F_{m^A} are detailed in Lemma 2.

1. *The equilibrium price of stock 1 is:*

$$\begin{aligned} S_1(t) &:= S_1\left(D_1, m^A, \Psi\right), \\ &= D_1(t)\int_t^\infty e^{-\delta(u-t)}F_{m^A}(m^A,t,u;1-\gamma,0)G\left(t,u,1-\gamma,0;,\Psi\right)du. \end{aligned}$$

2. The equilibrium price of stock 2 is:

$$\begin{aligned} S_2(t) &:= S_2\left(D_1, D_2, m^A, \Psi\right), \\ &= D_2(t)\int_t^{\infty} e^{-\delta(u-t)} F_{m^A}\left(m^A, t, u; -2\gamma, 1+\gamma\right) G\left(t, u, -2\gamma, 1+\gamma; , \Psi\right) du. \end{aligned}$$

3. The equilibrium price of the index is:

$$ID(t) := ID\left(D_1, D_2, m^A, \Psi\right) = \omega_1 S_1(t) + \omega_2 S_2(t).$$

4. The equilibrium price of the European option on stock 1 is:

$$\begin{aligned} O_1(t) &:= O_1\left(D_1, m^A, \Psi\right), \\ &= E_t^A\left(e^{-\delta(T-t)}\left(\frac{D_1(t)}{D_1(T)}\frac{1+\lambda(T)^{1/\gamma}}{1+\lambda(t)^{1/\gamma}}\right)^{\gamma}\left(S_1(T) - K_1\right)^+\right). \end{aligned}$$

The formula for the option on stock 2 is identical with the corresponding replacements, and with $S_2(T)$ and K_2 replacing $S_1(T)$ and K_1, respectively.

5. The equilibrium price of the European option on the index is:

$$\begin{aligned} I(t) &:= I\left(D_1, D_2, m^A, \Psi\right), \\ &= E_t^A\left(e^{-\delta(T-t)}\left(\frac{D_1(t)}{D_1(T)}\frac{1+\lambda(T)^{1/\gamma}}{1+\lambda(t)^{1/\gamma}}\right)^{\gamma}\left(ID(T) - K_{ID}\right)^+\right). \end{aligned}$$

2.6 Stock Price Volatility, Correlation and Skewness

The price of the stock satisfies a diffusion process which is given by:

$$\frac{dS_1}{S_1} = \mu_{S_1}^A(t)dt + \sigma_{S_1 D_1}(t)dW_{D_1}^A(t) + \sigma_{S_1 D_2}(t)dW_{D_2}^A(t) + \sigma_{S_1 z}(t)dW_z^A(t).$$

The diffusion term is characterized by:

$$
\begin{aligned}
dS_1(t) - S_1(t)\mu^A_{S_1}(t)dt &= \frac{\partial S_1}{\partial D_1}\left(dD_1(t) - E^A_t\left(dD_1(t)\right)\right) + \frac{\partial S_1}{\partial m^A_{D_1}}\left(dm^A_{D_1}(t) - E^A_t\left(dm^A_{D_1}\right)\right) + \frac{\partial S_1}{\partial \Psi_{D_1}}\left(d\Psi_{D_1}(t) - E^A_t\left(d\Psi_{D_1}(t)\right)\right) \\
&\quad + \frac{\partial S_1}{\partial \Psi_{D_2}}\left(d\Psi_{D_2}(t) - E^A_t\left(d\Psi_{D_2}(t)\right)\right) + \frac{\partial S_1}{\partial \Psi_z}\left(d\Psi_z - E^A_t\left(d\Psi_z(t)\right)\right), \\
&= \frac{\partial S_1}{\partial D_1} D_1 \sigma_{D_1} dW^A_{D_1}(t) \\
&\quad + \frac{\partial S_1}{\partial m^A_{D_1}}\left(\frac{\gamma^A_{D_1}}{\sigma_{D_1}} dW^A_{D_1}(t) + \frac{\gamma^A_{D_1D_2}}{\sigma_{D_2}} dW^A_{D_2}(t) + \left(\frac{\alpha_{D_1}\gamma^A_{D_1} + \alpha_{D_2}\gamma^A_{D_1D_2} + \beta\gamma^A_{D_1z}}{\sigma_z}\right) dW^A_z(t)\right) \\
&\quad + \frac{\partial S_1}{\partial \Psi_{D_1}}\Bigg(\left(\frac{\gamma^A_{D_1} - \gamma^B_{D_1}}{\sigma^2_{D_1}}\right) dW^A_{D_1}(t) + \left(\frac{\gamma^A_{D_1D_2} - \gamma^A_{D_1D_2}}{\sigma_{D_1}\sigma_{D_2}}\right) dW^A_{D_2}(t) \\
&\quad + \left(\frac{\alpha_{D_1}\left(\gamma^A_{D_1} - \gamma^B_{D_1}\right) + \alpha_{D_2}\left(\gamma^A_{D_1D_2} - \gamma^B_{D_1D_2}\right) + \beta\left(\gamma^A_{D_1z} - \gamma^B_{D_1z}\right)}{\sigma_{D_1}\sigma_z}\right) dW^A_z(t)\Bigg) \\
&\quad + \frac{\partial S_1}{\partial \Psi_{D_2}}\Bigg(\left(\frac{\gamma^A_{D_1D_2} - \gamma^B_{D_1D_2}}{\sigma_{D_1}\sigma_{D_2}}\right) dW^A_{D_1}(t) + \left(\frac{\gamma^A_{D_2} - \gamma^B_{D_2}}{\sigma^2_{D_2}}\right) dW^A_{D_2}(t) \\
&\quad + \left(\frac{\alpha_{D_1}\left(\gamma^A_{D_1D_2} - \gamma^B_{D_1D_2}\right) + \alpha_{D_2}\left(\gamma^A_{D_2} - \gamma^B_{D_2}\right) + \beta\left(\gamma^A_{D_2z} - \gamma^B_{D_2z}\right)}{\sigma_{D_2}\sigma_z}\right) dW^A_z(t)\Bigg) \\
&\quad + \frac{\partial S_1}{\partial \Psi_z}\Bigg(\left(\frac{\gamma^A_{D_1z} - \gamma^B_{D_1z}}{\sigma_{D_1}\sigma_z}\right) dW^A_{D_1}(t) + \left(\frac{\gamma^A_{D_2z} - \gamma^B_{D_2z}}{\sigma_{D_2}\sigma_z}\right) dW^A_{D_2}(t) \\
&\quad + \left(\frac{\alpha_{D_1}\left(\gamma^A_{D_1z} - \gamma^B_{D_1z}\right) + \alpha_{D_2}\left(\gamma^A_{D_2z} - \gamma^B_{D_2z}\right) + \beta\left(\gamma^A_z - \gamma^B_z\right)}{\sigma^2_z}\right) dW^A_z(t)\Bigg)
\end{aligned}
$$

where

$$
\begin{aligned}
\sigma_{S_1D_1}(t) &= \frac{1}{S_1(t)}\left(\frac{\partial S_1}{\partial D_1} D_1\sigma_{D_1} + \frac{\partial S_1}{\partial m^A_{D_1}}\frac{\gamma^A_{D_1}}{\sigma_{D_1}} + \frac{\partial S_1}{\partial \Psi_{D_1}}\left(\frac{\gamma^A_{D_1} - \gamma^B_{D_1}}{\sigma^2_{D_1}}\right) + \frac{\partial S_1}{\partial \Psi_{D_2}}\left(\frac{\gamma^A_{D_1D_2} - \gamma^B_{D_1D_2}}{\sigma_{D_1}\sigma_{D_2}}\right) + \frac{\partial S_1}{\partial \Psi_z}\left(\frac{\gamma^A_{D_1z} - \gamma^B_{D_1z}}{\sigma_{D_1}\sigma_z}\right)\right), \\
\sigma_{S_1D_2}(t) &= \frac{1}{S_1(t)}\left(\frac{\partial S_1}{\partial m^A_{D_1}}\frac{\gamma^A_{D_1D_2}}{\sigma_{D_2}} + \frac{\partial S_1}{\partial \Psi_{D_1}}\left(\frac{\gamma^A_{D_1D_2} - \gamma^B_{D_1D_2}}{\sigma_{D_1}\sigma_{D_2}}\right) + \frac{\partial S_1}{\partial \Psi_{D_2}}\left(\frac{\gamma^A_{D_2} - \gamma^B_{D_2}}{\sigma^2_{D_2}}\right) + \frac{\partial S_1}{\partial \Psi_z}\left(\frac{\gamma^A_{D_2z} - \gamma^B_{D_2z}}{\sigma_{D_2}\sigma_z}\right)\right), \\
\sigma_{S_1z}(t) &= \frac{1}{S_1(t)}\Bigg(\frac{\partial S_1}{\partial m^A_{D_1}}\left(\frac{\alpha_{D_1}\gamma^A_{D_1} + \alpha_{D_2}\gamma^A_{D_1D_2} + \beta\gamma^A_{D_1z}}{\sigma_z}\right) \\
&\quad + \frac{\partial S_1}{\partial \Psi_{D_1}}\left(\frac{\alpha_{D_1}\left(\gamma^A_{D_1} - \gamma^B_{D_1}\right) + \alpha_{D_2}\left(\gamma^A_{D_1D_2} - \gamma^B_{D_1D_2}\right) + \beta\left(\gamma^A_{D_1z} - \gamma^B_{D_1z}\right)}{\sigma_{D_1}\sigma_z}\right) \\
&\quad + \frac{\partial S_1}{\partial \Psi_{D_2}}\left(\frac{\alpha_{D_1}\left(\gamma^A_{D_1D_2} - \gamma^B_{D_1D_2}\right) + \alpha_{D_2}\left(\gamma^A_{D_2} - \gamma^B_{D_2}\right) + \beta\left(\gamma^A_{D_2z} - \gamma^B_{D_2z}\right)}{\sigma_{D_2}\sigma_z}\right) \\
&\quad + \frac{\partial S_1}{\partial \Psi_z}\left(\frac{\alpha_{D_1}\left(\gamma^A_{D_1z} - \gamma^B_{D_1z}\right) + \alpha_{D_2}\left(\gamma^A_{D_2z} - \gamma^B_{D_2z}\right) + \beta\left(\gamma^A_z - \gamma^B_z\right)}{\sigma^2_z}\right)\Bigg).
\end{aligned}
$$

We can now compute the stock volatility: $\left(\sigma^2_{S_1D_1}+\sigma^2_{S_1z_1}+\sigma^2_{S_1D_2}+\sigma^2_{S_1z_2}\right)^{1/2}$ using the formula for the Laplace transform. The corresponding coefficients for the volatility of stock 2 are:

$$\begin{aligned}
\sigma_{S_2D_1}(t) &= \frac{1}{S_2(t)}\left(\frac{\partial S_2}{\partial m^A_{D_2}}\left(\frac{\gamma^A_{D_1D_2}}{\sigma_{D_1}}\right)+\frac{\partial S_2}{\partial \Psi_{D_1}}\left(\frac{\gamma^A_{D_1}-\gamma^B_{D_1}}{\sigma^2_{D_1}}\right)+\frac{\partial S_2}{\partial \Psi_{D_2}}\left(\frac{\gamma^A_{D_1D_2}-\gamma^B_{D_1D_2}}{\sigma_{D_1}\sigma_{D_2}}\right)+\frac{\partial S_2}{\partial \Psi_{z}}\left(\frac{\gamma^A_{D_1z}-\gamma^B_{D_1z}}{\sigma_{D_1}\sigma_{z}}\right)\right),\\
\sigma_{S_2D_2}(t) &= \frac{1}{S_2(t)}\left(\frac{\partial S_2}{\partial D_2}D_2\sigma_{D_2}+\frac{\partial S_2}{\partial m^A_{D_2}}\frac{\gamma^A_{D_2}}{\sigma_{D_2}}+\frac{\partial S_2}{\partial \Psi_{D_1}}\left(\frac{\gamma^A_{D_1D_2}-\gamma^B_{D_1D_2}}{\sigma_{D_1}\sigma_{D_2}}\right)+\frac{\partial S_2}{\partial \Psi_{D_2}}\left(\frac{\gamma^A_{D_2}-\gamma^B_{D_2}}{\sigma^2_{D_2}}\right)+\frac{\partial S_2}{\partial \Psi_{z}}\left(\frac{\gamma^A_{D_2z}-\gamma^B_{D_2z}}{\sigma_{D_2}\sigma_{z}}\right)\right),\\
\sigma_{S_2z}(t) &= \frac{1}{S_2(t)}\left(\frac{\partial S_2}{\partial m^A_{D_2}}\left(\frac{\alpha_{D_1}\gamma^A_{D_1D_2}+\alpha_{D_2}\gamma^A_{D_2}+\beta\gamma^A_{D_2z}}{\sigma_z}\right)\right.\\
&\quad+\frac{\partial S_2}{\partial \Psi_{D_1}}\left(\frac{\alpha_{D_1}\left(\gamma^A_{D_1}-\gamma^B_{D_1}\right)+\alpha_{D_2}\left(\gamma^A_{D_1D_2}-\gamma^B_{D_1D_2}\right)+\beta\left(\gamma^A_{D_1z}-\gamma^B_{D_1z}\right)}{\sigma_{D_1}\sigma_z}\right)\\
&\quad+\frac{\partial S_2}{\partial \Psi_{D_2}}\left(\frac{\alpha_{D_1}\left(\gamma^A_{D_1D_2}-\gamma^B_{D_1D_2}\right)+\beta\left(\gamma^A_{D_2z}-\gamma^B_{D_2z}\right)}{\sigma_{D_2}\sigma_z}\right)\\
&\quad\left.+\frac{\partial S_2}{\partial \Psi_{z}}\left(\frac{\alpha_{D_1}\left(\gamma^A_{D_1z}-\gamma^B_{D_1z}\right)+\alpha_{D_2}\left(\gamma^A_{D_2z}-\gamma^B_{D_2z}\right)+\beta\left(\gamma^A_{z}-\gamma^B_{z}\right)}{\sigma^2_z}\right)\right).
\end{aligned}$$

The correlation between stock 1 and stock 2 can be calculated as follows:

$$\operatorname{corr}\left(\frac{dS_1}{S_1}\frac{dS_2}{S_2}\right)=\frac{Cov\left(\frac{dS_1}{S_1}\frac{dS_2}{S_2}\right)}{\sqrt{\sigma^2_{S_1}(t)}\sqrt{\sigma^2_{S_2}(t)}},$$

where

$$\begin{aligned}
Cov\left(\frac{dS_1}{S_1}\frac{dS_2}{S_2}\right) &= E\left(\frac{dS_1}{S_1}\frac{dS_2}{S_2}\right)-E\left(\frac{dS_1}{S_1}\right)E\left(\frac{dS_2}{S_2}\right),\\
&= \left(\sigma_{S_1D_1}(t)\sigma_{S_2D_1}(t)+\sigma_{S_1z}(t)+\sigma_{S_1D_2}(t)\sigma_{S_2D_2}(t)+\sigma_{S_2z}(t)\right)dt.
\end{aligned}$$

To synthesize the risk-neutral skewness, we follow Bakshi and Madan (2000) who show that the entire collection of twice-differentiable payoff functions with bounded expectation can be spanned algebraically. Applying this result to the stock price $S_1(t)$, we get

$$G(S_1)=G(\tilde{S}_1)+\left(S-\tilde{S}\right)G_{S_1}\left(\tilde{S}\right)+\int_{\tilde{S}_1}^{\infty}G_{S_1S_1}(K)\left(S_1-K\right)^+dK+\int_0^{\tilde{S}_1}G_{S_1S_1}(K)\left(K-S_1\right)^+dK,$$

where G_{S_1} is the partial derivative of the payoff function $G(S_1)$ with respect to S_1 and $G_{S_1S_1}$ the corresponding second-order partial derivative. By setting $\tilde{S}_1=S_1(t)$, we obtain the final formula for the risk-neutral skewness of the stock, after mimicking the steps in Bakshi, Kapadia, and Madan (2003) (Theorem 1, p. 137).

The risk-neutral skewness of stock S_1 is given by:

$$skew_{S_1}(t,T)=\frac{e^{rT}W(t,T)-2\mu(t,T)e^{rT}R(t,T)+2\mu(t,T)^3}{\left(e^{rT}R(t,T)-\mu(t,T)^2\right)^{3/2}},$$

where

$$
\begin{aligned}
R(t,T) &= \int_{S_1(t)}^{\infty} \frac{2\left(1-\ln\left(\frac{K}{S_1(t)}\right)\right)}{K^2}\left(S_1(K)-K\right)^{+} + \int_{0}^{S_1(t)} \frac{2\left(1+\ln\left(\frac{S_1(t)}{K_1}\right)\right)}{K^2}\left(K-S_1(T)\right)^{+} dK, \\
W(t,T) &= \int_{S_1(t)}^{\infty} \frac{6\ln\left(\frac{K}{S_1(t)}\right)-3\left(\ln\left(\frac{K}{S_1(t)}\right)\right)^2}{K^2}\left(S_1(t)-K\right)^{+} dK \\
&\quad - \int_{0}^{S_1(t)} \frac{6\ln\left(\frac{S_1(t)}{K}\right)-3\left(\ln\left(\frac{S_1(t)}{K}\right)\right)^2}{K^2}\left(K-S_1(T)\right)^{+} dK,
\end{aligned}
$$

and

$$
\begin{aligned}
X(t,T) &= \int_{S_1(t)}^{\infty} \frac{12\left(\ln\left(\frac{K}{S_1(t)}\right)\right)^2-4\left(\ln\left(\frac{K}{S_1(t)}\right)\right)^3}{K^2}\left(S_1(T)-K\right)^{+} dK \\
&\quad - \int_{0}^{S_1(t)} \frac{12\left(\ln\left(\frac{S_1(t)}{K}\right)\right)^2-4\left(\ln\left(\frac{S_1(t)}{K}\right)\right)^3}{K^2}\left(K-S_1(T)\right)^{+} dK, \\
\mu(t,T) &= E_t\left(\ln\left(\frac{S_1(t+T)}{S_1(t)}\right)\right) \approx e^{rT}-1-\frac{e^{rT}}{2}R(t,T)-\frac{e^{rT}}{6}W(t,T)-\frac{e^{rT}}{24}X(t,T).
\end{aligned}
$$

The skewness of stock 2 and of the index have similar expressions.

3 Additional Tables

This section contains additional tables that were omitted in the paper to save space.

3.1 Parameter Values

Table 1 reports the parameter values used throughout the paper for our calibration and simulations. The parameters of the fundamental process D_i are set to match the average growth rate and volatility of the S&P 500. The parameters choices for the disagreement processes match the moments of the average disagreement in the data. For example, the average disagreement about future earnings is around 4.9% which corresponds to the 5.4% average disagreement at the steady-state in our calibration. And the volatility of 7% corresponds roughly to the average earnings volatility in the data.

4 Robustness Checks

The economic and statistical significance of disagreement proxies is robust to the inclusion of different other control variables. In this section, we report two alternative control variable specifications. In addition, we show that the economic significance of the trading strategies considered in the main text is robust to the inclusion of transaction costs.

4.1 Volatility and Correlation Risk Premia

4.1.1 Different Measures of the Volatility Risk Premium

In our empirical exercise (see Section IV in the main text), we proxy the volatility risk premium as the difference between the expected implied volatility at time t and the expected realized volatility at time $t+1$. This definition differs from the one used by some other empirical studies which usually proxy the volatility risk premium as the difference between the ex-post realized return variation in t and the ex-ante risk neutral expectation of the future return variation over the t and $t+1$ time interval (see e.g., Bollerslev, Tauchen, and Zhou, 2010). In order to test, for robustness of our results with respect to an ex post measure of the volatility risk premium, we run the same panel regressions as in Section IV A of the paper. The results are gathered in Table 2.

[Insert Table 2 approximately here.]

Table 2 shows that both the economic and statistical significance of firm specific and common disagreement prevail when we use different proxies of the volatility risk premium. The magnitude of the estimated coefficient remains virtually the same for all factors. We conclude that a change in the definition of the volatility risk premium does not alter the results.

4.2 Fundamental Uncertainty and Earning Announcements

When firms announce earnings every quarter, they reveal firm fundamentals which were - to some extent - unknown to investors prior to the announcement. This uncertainty might be related to investor's expectations about future firm fundamentals, such as earnings. Empirical evidence has shown that volatility risk premia tend to be high prior to an earnings announcement. It is therefore an interesting question, whether the results implied by our proxy of belief disagreement are affected by the introduction of an uncertainty measure.

Ederington and Lee (1996) and Beber and Brandt (2006) document a strong decrease in implied volatility subsequent to major macroeconomic announcements in U.S. Treasury bond futures. While the first authors document that the implied volatility falls around announcements, the latter find in addition that skewness and kurtosis of the options returns distribution change after announcements. Dubinsky and Johannes (2006) find the same effect for earning announcements: Implied volatilities of single stock options increase prior to, and decrease subsequent to, an earning announcement. In particular, the risk-neutral volatility of price jumps deriving from earning announcements, which captures the anticipated uncertainty on the stock price embedded in an announcement, should be a priced risk factor. Using option prices, Dubinsky and Johannes (2006) develop an estimator of fundamental uncertainty surrounding announcement dates. They find no evidence of a price jump risk premium, but they find evidence of a jump volatility risk premium.[3] In the following, we estimate fundamental uncertainty using their term-structure estimator, defined as:[4]

$$\left(\sigma_{time}^{Q}\right)^{2} = T_i \left(\left(\sigma_{t,T_i}\right)^2 - \left(\sigma_{t+1,T_i-1}\right)^2 \right), \tag{12}$$

where σ_{t,T_i} is the Black-Scholes implied volatility at time t of an at-the-money option with $T_i - t$ days to maturity.

Frazzini and Lamont (2007) find that there is a premium around earnings announcement dates, which is large, robust, and related to the surging volume around these dates. A potential explanation for this finding is a surging difference of opinions about the meaning of the announcements (Kandel and Pearson, 1995). Since volume and returns move together during and after the announcement, the volume hypothesis can explain both the event-day returns and the post-event drift in returns.

[3] Evidence of a risk premium for volatility jumps in index options is also obtained by Broadie, Chernov, and Johannes (2008).

[4] This term-structure estimator is less noisy than a time-series estimator, as it does not depend on implied volatilities at different dates.

[Insert Table 3 approximately here.]

Fundamental Uncertainty: Fundamental uncertainty loads positively on the volatility risk premium, which is an intuitive finding: The higher the fundamental uncertainty, the larger the volatility risk premium. It is interesting that the estimated regression coefficient of belief disagreement is not affected by the inclusion of fundamental uncertainty. Moreover, the economic impact of belief disagreement on both individual and index volatility risk premia is larger than the one of fundamental uncertainty.

Earning Announcements: Quarterly earning announcement dates are from the I/B/E/S database. As noted in DellaVigna and Pollet (2009), before 1995 a large fraction of earnings announcements was recorded with an error of at least one trading day. In more recent years, the accuracy of the earnings date has increased substantially, and is almost perfect after December 1994. The variable earning announcement is a dummy variable having value of 1 if there was an earning announcement the previous month. The variable interaction is belief disagreement multiplied by this dummy variable. Earning announcements have a positive and highly significant impact on volatility risk premia, but there is no evidence of a significant interaction with disagreement. The inclusion of the additional variable does not affect the significance of the belief disagreement coefficient. Thus, belief disagreement has likely a significant impact on volatility risk premia independently of earning announcements.

4.3 Transaction Costs

Recent empirical evidence shows that transaction costs in option markets can be quite large and have a heavy impact on the profitability of option trading strategies.[5] We study the impact of bid and ask spreads on the performance of our trading strategies, by using bid returns when options are written and ask returns when options are bought. Results are reported in Table 4.

[Insert Table 4 approximately here.]

The results in Table 4 indicate that while indeed, the excess returns of both the straddle and dispersion trade drop from 0.16 to 0.10 and 0.06 to 0.04, respectively, the annualized Sharpe ratio is still attractive and approximately around 1. When running regressions from the excess returns on a set of risk factors, we find that the three Fama and French factors and momentum have virtually zero significance while the estimated coefficient for innovations in the common disagreement proxy are still significant. We conclude while transaction costs render both the straddle and dispersion trade less attractive, the economic significance of disagreement still prevails.

5 Dispersion Trading

Consider an index with n stocks. σ_i is the volatility of stock i, ω_i is the weight of stock i in the index, and ρ_{ij} is the correlation between stock i and stock j. The index return itself has the following variance:

$$\sigma^2_{Index} = \sum_{i=1}^{n} \omega_i^2 \sigma_i^2 + \sum_{i \neq j}^{n} \omega_i \omega_j \sigma_i \sigma_j \rho_{ij}.$$

[5] See e.g. Constantinides, Jackwerth, and Perrakis (2008), Santa-Clara and Saretto (2009) and Driessen, Maenhout, and Vilkov (2009).

Now, if ρ is equal to 1, the average index variance is:

$$\bar{\sigma}_{Index}^2 = \sum_{i=1}^{n} \omega_i \sigma_i^2.$$

We can now compare this number to the actual index volatility. We define a dispersion, D, as:

$$D = \sqrt{\bar{\sigma}_{Index}^2 - \sigma_{Index}^2}.$$

The upper bound for the dispersion is now simply the average basket volatility and the lower bound is zero. A trading strategy which bets on dispersion has two legs. If the investor is long dispersion, then she is long the volatility of the constituents and short index volatility. One of the main drivers of this strategy is the exposure to correlation. Considering the average correlation, it is easy to see that if one is long dispersion then she is short correlation:

$$\bar{\rho} = \frac{\sigma_{Index}^2 - \sum_{i=1}^{n} \omega_i^2 \sigma_i^2}{\sum_{i \neq j}^{n} \omega_i \omega_j \sigma_i \sigma_j}.$$

5.1 P&L of a Dispersion Trade

For simplicity, we study the case with constant volatility.[6] Consider a delta-hedged portfolio long the stock options and short the index options. Remember that the P&L of a delta-hedged option Π in a Black and Scholes framework is (see Hull, 2002)

$$P\&L = \theta \left[\left(\frac{dS}{\sigma \sqrt{dt}} \right)^2 - 1 \right],$$

where the θ is the option's sensitivity with respect to a change in the time to maturity.

Let the term $n = \frac{dS}{\sigma\sqrt{dt}}$ represent the standardized move of the underlying stock S on the considered time period. Then, the P&L of the index can be written as

$$\begin{aligned} P\&L &= \theta_I \left(n_I^2 - 1 \right), \\ &= \theta_I \left(\left(\sum_{i=1}^{n} \omega_i n_i \frac{\sigma_i}{\sigma_{Index}} \right)^2 - 1 \right), \\ &= \theta_I \left(\sum_{i=1}^{n} \left(\omega_i n_i \frac{\sigma_i}{\sigma_{Index}} \right)^2 + \sum_{i \neq j} \frac{\omega_i \omega_j \sigma_i \sigma_j}{\sigma_{Index}^2} n_i n_j - 1 \right), \\ &= \theta_I \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{\omega_i \omega_j \sigma_i \sigma_j}{\sigma_{Index}^2} \left(n_i n_j - \delta_{ij} \right), \end{aligned}$$

[6] Adding stochastic volatility yields analogous expressions with some additional terms which account for the Vega, Volga, and Vanna of the option.

where $i,j = 1$ for $i = j$ and 0 else. Hence, a dispersion trade being short the index options and being long the individual options has the following P&L:

$$\begin{aligned} P\&L &= \sum_{i=1}^{n} P\&L_i - P\&L_I, \\ &= \sum_{i=1}^{n} \theta_i \left(n_i^2 - 1\right) - \theta_I \left(n_I^2 - 1\right). \end{aligned}$$

The short and long positions in the options are reflected in the sign of the θ_i's. A long (short) position means a positive (negative) θ.

References

Bakshi, G., and D. Madan (2000): "Spanning and Derivative Security Valuation," *Journal of Financial Economics*, 55, p. 205 – 238. pages

Bakshi, G., N.Kapadia, and D. Madan (2003): "Stock Return Characteristics, Skew Laws, and the Differential Pricing of Individual Equity Options," *Review of Financial Studies*, 16, p. 101 – 143. pages

Basak, S. (2005): "Asset Pricing with Heterogeneous Beliefs," *Journal of Banking and Finance*, 29, p. 2849 – 2881. pages

Beber, A., and M. W. Brandt (2006): "The Effect of Macroeconomic News on Beliefs and Preferences: Evidence from the Options Market," *Journal of Monetary Economics*, 53, p. 1997 – 2039. pages

Bollerslev, T. G., G. Tauchen, and H. Zhou (2010): "Expected Stock Returns and Variance Risk Premia," *Review of Financial Studies*, 22, p. 4463 – 4492. pages

Britten-Jones, M., and A. Neuberger (2000): "Option Prices, Implied Price Processes, and Stochastic Volatility," *Journal of Finance*, 55, p. 839 – 866. pages

Broadie, M., M. Chernov, and M. Johannes (forthcoming, 2008): "Understanding Index Option Returns," *Review of Financial Studies*. pages

Carr, P., and D. Madan (1998): *Volatility: New Estimation Techniques for Pricing Derivatives*chap. Towards a Theory of Volatility Trading, pp. p. 417 – 427. RISK Publications. pages

Carr, P., and L. Wu (2009): "Variance Risk Premiums," *Review of Financial Studies*, 22, p. 1311 – 1341. pages

Constantinides, G. M., J. Jackwerth, and S. Perrakis (2008): "Mispricing of S&P 500 Index Options," *Review of Financial Studies*, 22, p. 1247 – 1277. pages

DellaVigna, S., and J. Pollet (2009): "Investor Inattention and Friday Earnings Announcements," *Journal of Finance*, 64, p. 709 – 749. pages

Demeterfi, K., E. Derman, M. Kamal, and J. Zhou (1999): "A Guide to Volatility and Variance Swaps," *Journal of Derivatives*, 6, p. 9 – 32. pages

Driessen, J., P. Maenhout, and G. Vilkov (2009): "The Price of Correlation Risk: Evidence from Equity Options," *Journal of Finance*, 64, p. 1377 – 1406. pages

Dubinsky, A., and M. Johannes (2006): "Earning Announcements and Equity Options," Working Paper, Columbia University. pages

Ederington, L., and J. H. Lee (1996): "The Creation and Resolution of Market Uncertainty: The Impact of Information Releases on Implied Volatility," *Journal of Financial and Quantitative Analysis*, 31, p. 513 – 539. pages

Forsythe, G. E., M. A. Malcolm, and C. B. Moler (1977): *Computer Methods for Mathematical Computations*. Prentice-Hall. pages

Frazzini, A., and O. Lamont (2007): "The Earnings Announcement Premium and Trading Volume," NBER Working Paper, No. 13090. pages

Hull, J. C. (2002): *Options, Futures, and Other Derivatives*. Prentice Hall, 5th edn. pages

Jiang, G., and Y. Tian (2005): "Model-Free Implied Volatility and Its Information Content," *Review of Financial Studies*, 18, p. 1305 – 1342. pages

Kandel, E., and N. Pearson (1995): "Differential Interpretation of Public Signals and Trade in Speculative Markets," *Journal of Political Economy*, 103, p. 831 – 872. pages

Newey, W. K., and K. D. West (1987): "A Simple, Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica*, 55, p. 703 – 708. pages

Pástor, L., and R. F. Stambaugh (2003): "Liquidity Risk and Expected Stock Returns," *Journal of Political Economy*, 111, p. 642 – 685. pages

Santa-Clara, P., and A. Saretto (2009): "Option Strategies: Good Deals and Margin Calls," *Journal of Financial Markets*, 12, p. 391 – 417. pages

Table 1: Choice of Parameter Values and Benchmark Values of State Variables

This table lists the parameter values used for all figures in the paper. We calibrate the model to the mean and volatility of the dividends on the S&P 500. The average growth rate for the period 1996-2006 is 5.93% and the volatility is 3.52%. The initial values for the conditional variances are set to their steady-state variances. We study a symmetric economy, where parameters for firm 1 and 2 are assumed to be the same.

PARAMETERS FOR FUNDAMENTALS		
Long-term growth rate of dividend growth	a_{0D_i}	0.01
Mean-reversion parameter of dividend growth	a_{1D_i}	-0.01
Volatility of dividend	σ_{D_i}	0.07
Initial level of dividend	D_i	1.00
Initial level of dividend growth	$m^A_{D_i}$	0.01
PARAMETERS FOR SIGNAL		
Long-term growth rate of signal	a_{0z}	0.01
Mean-reversion parameter of signal	a_{1z}	-0.03
Volatility of signal	σ_z	0.06
AGENT SPECIFIC PARAMETERS		
Relative risk aversion for both agents	γ	2.00
Time Preference Parameter	δ	0.02

Table 2: Volatility Risk Premium Regressions with Ex Post Volatility

Using data from January 1996 to June 2007, we run regressions of proxies for the difference between the index and individual volatility risk premium on a number of determinants. DiB is a proxy for difference in beliefs for each firm, defined as the mean absolute difference among analysts' forecasts. Common DiB is a proxy for the common component in the difference in beliefs, Market Vola is the 21 day realized volatility of the index, Skewness is the difference between the implied volatility of a put with 0.92 strike-to-spot ratio (or the closest available) and the implied volatility of an at-the-money put, dividend by the absolute difference in strike-to-spot ratios. CAPM Beta is estimated from a regression using a window of 180 daily returns. Liquidity is the ratio between trading volume and shares outstanding. DP is demand pressure and is defined as the difference between the number of contracts traded during the month at prices higher than the prevailing bid/ask quote midpoint and the number of contracts traded during the month at prices below the prevailing bid/ask quote midpoint, times the absolute value of the option's delta, then scaled by the total trading volume across all option series. Macro Factor is a dynamic factor from IP, Housing Starts, S&P 500, P/E ratio, and PPI. Sentiment is the first principal component from trading volume as measured by NYSE turnover, the dividend premium, the closed-end fund discount, the number and first-day returns on IPOs and the equity share in new issues. $\star$ denotes significance at the 10% level, $\star\star$ denotes significance at the 5% level and $\star\star\star$ denotes significance at the 1% level. All estimations use autocorrelation and heteroskedasticity-consistent t-statistics reported in parenthesis below the estimated coefficient.

		Individual		Index	
	(1)	(2)	(3)	(4)	(5)
Constant	0.001***	0.002**	0.001***	0.001***	0.001***
	(8.21)	(2.38)	(5.78)	(7.89)	(6.11)
DiB	0.037***	0.028***	0.042***		
	(22.18)	(8.93)	(19.82)		
Common DiB	0.012**	0.018**	0.010**	0.028***	0.038***
	(2.37)	(2.39)	(2.49)	(5.28)	(6.28)
Market Vola	0.028**		0.030**	0.097*	0.091**
	(2.45)		(1.99)	(1.83)	(2.47)
Macro Factor	−0.031**	−0.019*	−0.021**	−0.010**	−0.012*
	(-2.25)	(-1.93)	(-2.33)	(-2.29)	(-1.69)
Liquidity	−0.002*	−0.001*	-0.001		
	(-1.92)	(-1.88)	(-1.21)		
CAPM Beta		0.021*	0.018*		
		(1.72)	(1.77)		
Skewness	−0.012*	-0.010	-0.009	−0.014*	-0.011
	(-1.71)	(-1.50)	(-0.83)	(-1.89)	(-1.32)
DP Calls			−0.041		0.098
			(-1.61)		(1.02)
DP Puts			−0.020		0.029*
			(-1.29)		(1.71)
Adj. R^2	0.26	0.30	0.30	0.22	0.24

Table 3: Fundamental Uncertainty and Earning Announcements

Using data from January 1996 to June 2007, we run regressions from the volatility risk premium of individual and index options on a number of determinants. The volatility risk premium is defined as the difference between the optionsüf 21 day realized and implied volatility. DiB is our proxy for difference in beliefs for each firm, defined as the mean absolute difference among analysts forecasts standardized, Common DiB is a proxy for the common component in the difference in beliefs, Market Vola is the 21 day realized volatility of the index, Skewness is measured as the difference between the implied volatility of a put with 0.92 strike-to-spot ratio (or the closest available) and the implied volatility of an at-the-money put, dividend by the difference in strike-to-spot ratios. CAPM Beta is estimated from a regression using a window of 180 daily returns. Liquidity is the ratio between trading volume and shares outstanding. Earning Announcement is a dummy variable which takes the value of 1 if there is an earning announcement scheduled for the respective month and zero else. Interaction is the variable Earning Announcement multiplied by DiB. Fundamental Uncertainty is defined as in equation 12. DP is demand pressure and is defined as the difference between the number of contracts traded during the day at prices higher than the prevailing bid/ask quote midpoint and the number of contracts traded during the day at prices below the prevailing bid/ask quote midpoint, times the absolute value of the optionüfs delta and then scale this difference by the total trading volume across all option series. Macro Factor is a common component estimated via dynamic factor analysis from Industrial production, Housing Starts, S&P 500 P/E ratio, and, Producer Price index (PPI). We use logarithmic changes over the past twelve months. $\star$ denotes significance at the 10% level, $\star\star$ denotes significance at the 5% level and $\star\star\star$ denotes significance at the 1% level. All estimations use autocorrelation and heteroskedasticity-consistent t-statistics reported in parenthesis below the estimated coefficient.

	Individual		Index
Constant	0.001***	0.001***	0.001***
	(7.37)	(6.48)	(4.32)
DiB	0.043***	0.038***	
	(17.83)	(18.82)	
Common DiB	0.009**	0.019**	0.045***
	(2.21)	(2.53)	(5.37)
Market Vola	0.031**	0.029*	0.136**
	(1.99)	(1.83)	(2.33)
Macro Factor	−0.039**	−0.013**	−0.017***
	(-2.52)	(-2.39)	(-3.41)
Liquidity	−0.001*	-0.001	
	(-1.66)	(-1.33)	
CAPM Beta	0.019*	0.022*	
	(1.84)	(1.90)	
Skewness	-0.011	-0.008	-0.002
	(-1.12)	(-1.16)	(-1.17)
Earning Ann.	0.004***		
	(2.74)		
Interaction	0.018		
	(1.13)		
Funda. Uncer.			0.121***
			(3.29)
Adj. R^2	0.28	0.31	0.25

Table 4: Impact of Transaction Costs on Returns on Option Strategies

This table reports the monthly mean and annualized Sharpe ratio of the low minus high straddle and dispersion portfolio returns. Portfolios are sorted according to the size of each firm's individual DiB. Low (high) corresponds to quintile 1 (5) consists of stocks with the lowest (largest) DiB. The alpha and beta coefficients are estimated from the following least-squares regression:

$$exret_i(t) = \alpha_i + \beta_i^M MRKT(t) + \beta_i^S SMB(t) + \beta_i^{BM} HML(t) + \beta_i^{MOM} MOM(t) + \beta_i^V \epsilon^V(t) + \beta_i^L \epsilon^L(t) + \beta_i^S \epsilon^S(t) + \beta_i^D \epsilon^D(t) + \beta_i^C \epsilon^C(t) + u_i(t),$$

where $exret_i(t)$ is the strategy return in excess of the one month Libor, $MRKT$ is the value-weighted excess return on all NYSE, AMEX, and NASDAQ stocks, SMB is the size factor, HML is the book-to-market factor and MOM is the momentum factor. ϵ^V is the monthly change of the VIX, ϵ^L the monthly change of the aggregate liquidity measure, ϵ^D the change of the common disagreement factor, and ϵ^C the change in the realized correlation. $\star$ denotes significance at the 10% level, $\star\star$ denotes significance at the 5% level and $\star\star\star$ denotes significance at the 1% level. The t-statistics reported in parenthesis are based on Newey and West (1987) adjusted standard errors. We use bid prices when options are written and ask prices when options are bought. Option returns of single-stocks and the index are sampled between January 1996 and June 2007. All statistics are monthly, except the Sharpe ratios, which are annualized.

	MID POINT		BID-TO-ASK	
	Straddle	Dispersion	Straddle	Dispersion
Mean	0.1591	0.0616	0.1012	0.0422
Sharpe Ratio	1.3249	1.1034	1.0154	0.9987
Alpha	0.186*	0.027*	0.066*	0.013*
	(1.92)	(1.87)	(1.88)	(1.69)
MRKT	2.120	0.043	1.012	0.033
	(1.04)	(0.51)	(0.84)	(0.52)
SMB	-3.116	0.237	-2.458	0.214
	-(0.14)	(1.35)	-(0.42)	(1.48)
HML	0.106	-0.087	0.106	-0.096
	(0.94)	-(1.01)	(0.99)	-(0.84)
MOM	0.475	-0.093	0.501	-0.093
	(1.38)	-(0.87)	(1.44)	-(0.88)
ϵ^D	0.252**	0.699**	0.178**	0.700**
	(1.98)	(1.99)	(2.01)	(1.86)
ϵ^L	0.903	0.170	0.882	0.165
	(1.02)	(0.54)	(1.54)	(0.55)
ϵ^V	-1.430	0.658*	-1.457	0.656*
	-(0.55)	(1.79)	-(0.96)	(1.81)
ϵ^S	-0.015	1.456**	-0.023	1.234**
	-(1.03)	(2.39)	-(1.43)	(2.45)
ϵ^C	0.016*	0.006**	0.025*	0.006**
	(1.69)	(1.97)	(1.75)	(2.00)
Adj. R^2	0.08	0.06	0.06	0.05
Observations	132	132	132	132