For Online Publication

Estimation of Path Shocks

This section discusses different approaches to estimate monetary policy path shocks. For ease of notation, re-write the model for panel data as:

$$y_{nt} = x'_{nt}\beta + \alpha_n + \epsilon_{nt},$$

where

$$\begin{aligned}
y_{nt} &= f^e_{n,t,h} \\
x'_{nt} &= \begin{bmatrix} 1 & f^e_{n,t,h-1}, \dots f^e_{n,t,h-1}, & \pi^e_{n,t,h+j}, & x^e_{n,t,h+k} \end{bmatrix}' \\
\epsilon_{nt} &= u^e_{n,t,h};
\end{aligned}$$

α_n is an agent-specific term that controls for possible cross-sectional unobservable heterogeneity. A structural interpretation of α_n may be, for instance, that agents have different opinions about the long run mean of the federal funds rate, or, alternatively, about the inflation target of the Central Bank. The assumption that monetary policy shocks are orthogonal to the arguments of the Taylor rule implies:

$$E\left[\epsilon_{nt} | x_{nt}\right] = 0.$$

Unobserved heterogeneity, on the other hand, may be correlated (mean dependence) or not correlated (mean independence) with the arguments of the Taylor rule. The response coefficient β can be estimated from forecast data in at least four ways: (i) using consensus data, via OLS; or, using panel data, via: (ii) pooled OLS (POLS); (iii) fixed effects (FE); (iv) random effects (RE). It is natural to ask what are the relative advantages and disadvantages of each estimator, and under which conditions they provide the same answer.

Consider first the case of mean independence. Stack the N cross-sectional observations at time t into a single equation:

$$y_t = x_t\beta + \alpha + \epsilon_t;$$

the POLS estimator is given by:

$$\begin{aligned}
\hat{\beta}_P &= \left(\sum_{t=1}^{T} x'_t x_t\right)^{-1} \sum_{t=1}^{T} x'_t y_t \\
&= \left(\sum_{t=1}^{T} x'_t x_t\right)^{-1} \sum_{t=1}^{T} x'_t (x_t\beta + \alpha + \epsilon_t)
\end{aligned}$$

$$= \beta + \left(\sum_{t=1}^{T} x_t' x_t \right)^{-1} x_t' \alpha + \left(\sum_{t=1}^{T} x_t' x_t \right)^{-1} x_t' \epsilon_t.$$

Notice that the term

$$E \left[\left(\sum_{t=1}^{T} x_t' x_t \right)^{-1} x_t' \alpha \right]$$

is zero under the assumption of mean independence, and non-zero otherwise. Also, under the assumption $E[\epsilon_{nt}|x_{nt}] = 0$, an application of the law of iterated expectations implies:

$$\begin{aligned} E \left[\left(\sum_{t=1}^{T} x_t' x_t \right)^{-1} x_t' \epsilon_t \right] &= E \left[E \left[\left(\sum_{t=1}^{T} x_t' x_t \right)^{-1} x_t' \epsilon_t \middle| x_t \right] \right] \\ &= E \left[\left(\sum_{t=1}^{T} x_t' x_t \right)^{-1} x_t' E \left[\epsilon_t | x_t \right] \right] \\ &= 0. \end{aligned}$$

The implications is that the POLS estimator is unbiased only if mean independence holds; if, on the other hand, heterogeneity is correlated with the regressors, the POLS estimator is biased and a FE estimator should be used. Now let variables without n-subscripts and with over-bars denote cross-sectional averages, and consider the consensus model:

$$\bar{y}_t = \bar{x}_t \beta + \bar{\alpha} + \bar{\epsilon}_t;$$

By following the same steps outlined above, it can be shown that the consensus estimator can be written as:

$$\hat{\beta}_C = \beta + \left(\sum_{t=1}^{T} \bar{x}_t' \bar{x}_t \right)^{-1} \bar{x}_t' \bar{\alpha} + \left(\sum_{t=1}^{T} \bar{x}_t' \bar{x}_t \right)^{-1} \bar{x}_t' \bar{\epsilon}_t.$$

Once more, the assumption of mean independence implies:

$$E \left[\left(\sum_{t=1}^{T} \bar{x}_t' \bar{x}_t \right)^{-1} \bar{x}_t' \bar{\alpha} \right] = 0;$$

however, we have that:

$$E \left[\left(\sum_{t=1}^{T} \bar{x}_t' \bar{x}_t \right)^{-1} \bar{x}_t' \bar{\epsilon}_t \right] = E \left[E \left[\left(\sum_{t=1}^{T} \bar{x}_t' \bar{x}_t \right)^{-1} \bar{x}_t' \bar{\epsilon}_t \middle| \bar{x}_t \right] \right]$$

$$= E\left[\left(\sum_{t=1}^{T}\bar{x}_t'\bar{x}_t\right)^{-1}\bar{x}_t'E\left[\bar{\epsilon}_t|\,\bar{x}_t\right]\right]$$
$$\neq 0,$$

implying that, even under the assumptions of mean independence, the consensus estimator is biased unless

$$E\left[\bar{\epsilon}_t|\bar{x}_t\right] = E\left[\epsilon_{nt}|x_{nt}\right] = 0.$$

Should this condition hold, the consensus estimator would be unbiased, albeit inefficient compared to the pooled OLS estimator.

In summary, the choice of the estimator depends on the assumptions about the orthogonality between cross-sectional average of the forecasts about the arguments of the Taylor rule and the cross-sectional average of monetary shocks forecasts, on one hand, and unobserved heterogeneity, on the other hand. Absent economic priors, it is possible to dispense with the need to make an assumption about either condition, and rely on a purely statistical criteria for the selection of the estimator. In particular, the null hypothesis of no fixed effects can be tested by running an F-test on the hypothesis of joint significance of the agent dummies. The null hypothesis of random effects, on the other hand, can be tested by means of a Hausman test.

Additional Figures

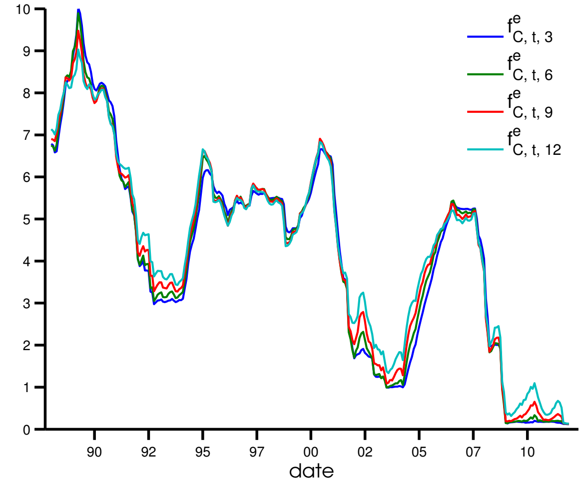

Figure 7. Federal Funds Rate Forecasts:
Figure plots 1-quarter to 4-quarter consensus forecasts for the level of the Federal funds rate in percentage points.

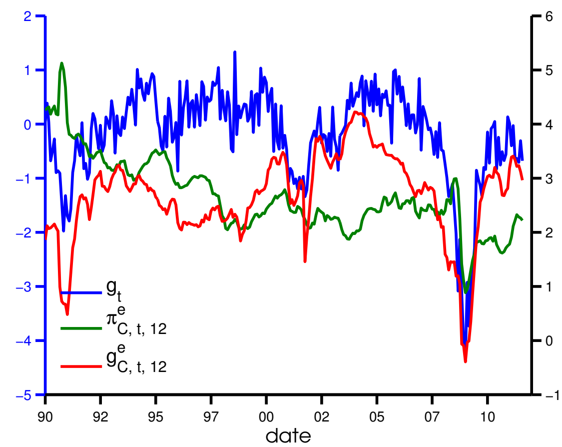

Figure 8. Consensus Macro Forecasts:
This figure plots the time series of consensus forecasts for 1-year inflation ($\pi^e_{C,t,12}$), GDP growth ($g^e_{C,t,12}$) and proxy for the level of macroeconomic activity (g_t) described in the data section. Inflation and GDP forecasts are plotted against the right y-axis in percentage points. Macroeconomic activity is plotted on the left axis in standardised units.

Additional Tables

Robustness Tests

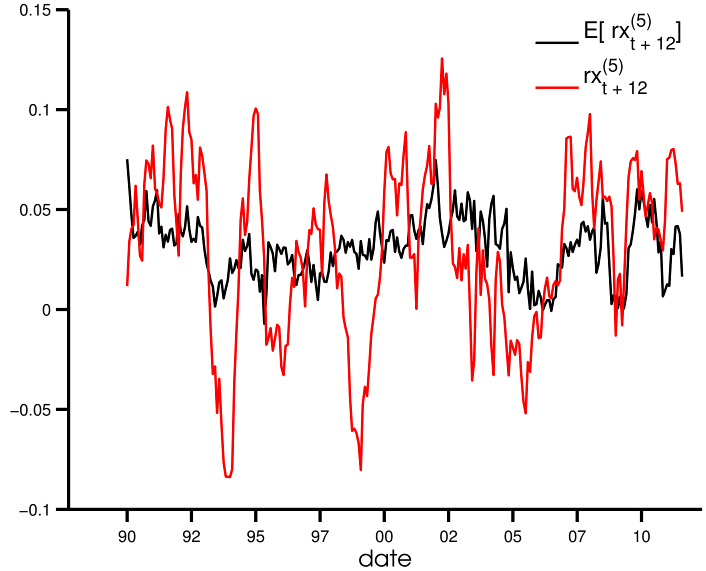

Figure 9. $\mathcal{P}athShock$ **and excess returns**
This figure plots expected excess returns on 5-year bonds conditional on $\mathcal{P}athShock$ versus realised excess returns. Sample period: 1990:1 - 2011:7.

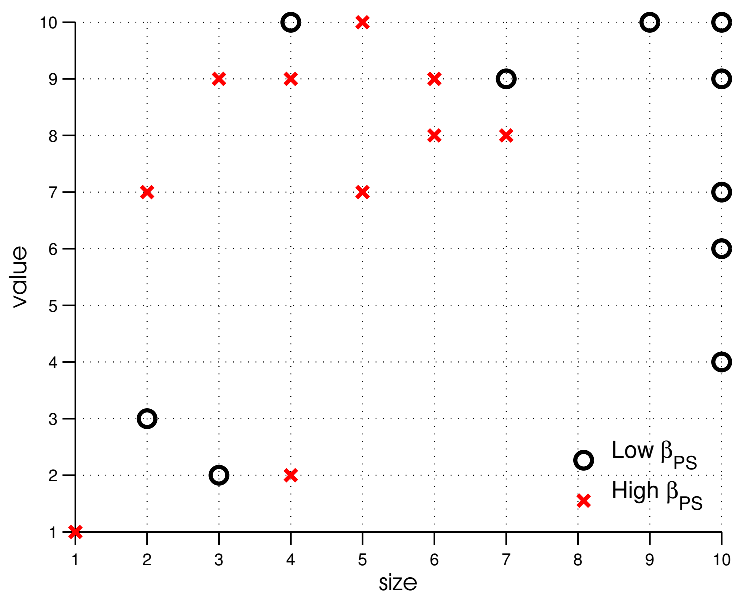

Figure 10. Constituents of *mps*
This figure illustrates the size and value characteristics of the test assets used to construct the mimicking portfolio for monetary policy shocks (specification 1). Black circles (red crosses) indicate the assets with the lowest (highest) sensitivity to monetary policy shocks.

Table XII. Summary Statistics for Bond Excess Returns

This table presents the summary statistics of one-year bond excess returns constructed from log-prices of Fama-Bliss zero coupon bonds:

$$rx_{t+12}^{(n)} = p_{t+12}^{(n-1)} - p_t^{(n)} - y_t^{(1)}.$$

Bond maturities n range from 2 to 5 years. Sample period: 1990:1 - 2011:7 (last realised excess return is between 2011:7 and 2012:7). Number of observations is 260.

Maturity	2 Year	3 Year	4 Year	5 Year
Mean	0.94	1.81	2.58	3.07
Std Dev	1.34	2.54	3.56	4.41
Min	−2.37	−5.25	−6.90	−8.39
Max	3.63	7.33	10.32	12.56
Skew	−0.11	−0.28	−0.37	−0.45
Kurtosis	2.22	2.51	2.58	2.68
1st Lag Auto	0.95	0.94	0.93	0.92
2nd Lag Auto	0.88	0.86	0.84	0.82
3rd Lag Auto	0.80	0.77	0.74	0.71

Table XIII. Summary statistics for $\mathcal{P}ath\mathcal{S}hock$

This table shows the summary statistics of monetary policy path shocks $\mathcal{P}ath\mathcal{S}hock_t^i$, constructed as cross-sectional averages of the residuals from Taylor rules estimated over a panel of forecast data. Each series (i) correspond to one of the 6 specifications described in Table I. Sample period: 1990:1 - 2011:7.

Spec	**1**	**2**	**3**	**4**	**5**	**6**
Observations	260	260	260	260	260	260
Mean	0.39%	0.38%	0.35%	0.35%	0.37%	0.37%
Std Dev	4.61%	4.55%	5.58%	5.63%	5.51%	5.60%
Min	-10.59%	-10.68%	-10.60%	-11.04%	-11.72%	-12.33%
Max	14.21%	13.41%	14.80%	14.84%	14.69%	14.98%
Skew	0.10	-0.02	0.34	0.27	0.33	0.30
Kurtosis	2.76	2.68	2.37	2.39	2.44	2.46
1st lag AC	0.80	0.79	0.86	0.86	0.85	0.85
2nd lag AC	0.67	0.66	0.73	0.73	0.72	0.72
3rd lag AC	0.59	0.57	0.62	0.62	0.61	0.62

Table XIV. Macroeconomic uncertainty

This table shows the point estimates and t-statistics of an ARMA(1,1)-GARCH(1, 1) model for consensus expectations about 1-year-ahead GDP and inflation.

	ARMA(1, 1)			GARCH(1, 1)		
	const	AR	MA	const.	GARCH	ARCH
$g^e_{C,t,1Y}$	0,04	0,98	0,31	0,00	0,84	0,16
	0,50	35,03	5,78	2,17	29,05	7,01
$\pi^e_{C,t,1Y}$	0,11	0,96	0,46	0,00	0,43	0,57
	3,66	89,62	6,72	2,98	5,43	6,77

Table XV. Excess Returns, Habit and LRR

The table reports the output from regressions of annual bond excess returns on proxies (x_t) of consumption surplus and long run risk:

$$rx^{(n)}_{t+12} = const. + \beta^{(n)} x_t + \epsilon^{(n)}_{t+12}$$

Bond maturities (n) range from 2 to 5 years. The proxies are: consumption surplus (s), GDP growth uncertainty ($\sigma(g^e)$), inflation uncertainty ($\sigma(\pi^e)$).T-statistics, reported below the point estimates, are corrected for auto-correlation and heteroskedasticity using Newey-West errors (18 lags). $\bar{R}^2$ is the adjusted R^2. Both left and right hand variables are standardized. A constant is included but not reported.

	Habit		LRR			Habit and LRR			
n	s	$\bar{R}^2$	$\sigma(g^e)$	$\sigma(\pi^e)$	$\bar{R}^2$	s	$\sigma(g^e)$	$\sigma(\pi^e)$	$\bar{R}^2$
2	-0,08	0,22%	0,37	0,04	14,94%	0,04	0,38	0,05	14,76%
	(-0,46)		(3,03)	(0,53)		(0,26)	(3,75)	0,60	
3	-0,09	0,39%	0,41	0,01	16,83%	0,03	0,42	0,01	16,61%
	(-0,50)		(3,64)	(0,09)		(0,24)	(4,34)	(0,14)	
4	-0,14	1,47%	0,42	-0,04	16,13%	-0,03	0,42	-0,04	15,87%
	(-0,87)		(3,82)	(-0,42)		(-0,20)	(4,15)	(-0,45)	
5	-0,17	2,46%	0,41	-0,05	15,08%	-0,07	0,40	-0,05	15,16%
	(-1,34)		(3,98)	(-0,62)		(-0,53)	(4,00)	(-0,71)	

Table XVI. Bond return predictability: $\mathcal{P}ath\mathcal{S}hock$

The table reports the output from regressions of annual bond excess returns on a constant and expected monetary policy shocks:

$$rx_{t+12}^{(n)} = const. + \beta_{PS}^{(n)} \mathcal{P}ath\mathcal{S}hock_t + \epsilon_{t+12}^{(n)}.$$

Bond maturities (n) range from 2 to 5 years. Each panel reports the results for one of the 6 proxies of expected monetary policy described by Table I. The left panels report the results for the full sample (the last observation is the excess return that realized between 2011:7 and 2012:7), while the right panels report the results for the sample excluding the crisis (the last observation is the excess return that realized between 2007:6 and 2008:6). T-statistics, reported below the point estimates, are corrected for auto-correlation and heteroskedasticity using Newey-West errors (18 lags). $\bar{R}^2$ is the adjusted R^2. Both left and right hand variables are standardized. A constant is included but not reported.

	1990-2011		1990-2007	
n	$\mathcal{P}ath\mathcal{S}hock$	$\bar{R}^2$	$\mathcal{P}ath\mathcal{S}hock$	$\bar{R}^2$
	Specification 1			
2	0.43	17.88%	0.43	18.18%
	3.88		3.61	
3	0.42	16.93%	0.42	16.84%
	3.85		3.48	
4	0.41	16.18%	0.40	15.62%
	3.74		3.30	
5	0.37	13.35%	0.37	13.37%
	3.40		3.08	
	Specification 2			
2	0.38	14.00%	0.40	15.53%
	3.13		3.07	
3	0.37	13.73%	0.39	14.62%
	3.19		3.00	
4	0.38	14.05%	0.38	13.70%
	3.35		2.89	
5	0.35	12.05%	0.35	12.00%
	3.19		2.77	

Table XVII. Bond return predictability: $\mathcal{P}athShock$

continued from above.

	1990-2011		1990-2007	
n	$\mathcal{P}athShock$	$\bar{R}^2$	$\mathcal{P}athShock$	$\bar{R}^2$
		Specification 3		
2	0.37	13.56%	0.40	15.95%
	2.97		3.03	
3	0.36	12.83%	0.41	16.09%
	2.92		3.05	
4	0.35	11.81%	0.41	16.07%
	2.78		3.03	
5	0.31	9.38%	0.39	14.53%
	2.50		2.95	
		Specification 4		
2	0.36	12.54%	0.39	14.95%
	2.76		2.80	
3	0.35	12.10%	0.40	15.30%
	2.76		2.84	
4	0.35	11.72%	0.40	15.41%
	2.79		2.86	
5	0.32	9.69%	0.38	14.24%
	2.62		2.82	
		Specification 5		
2	0.35	11.95%	0.38	13.82%
	2.71		2.68	
3	0.34	11.36%	0.38	14.01%
	2.64		2.69	
4	0.33	10.30%	0.38	13.97%
	2.50		2.67	
5	0.29	8.01%	0.36	12.49%
	2.22		2.57	

Table XVIII. Bond return predictability: $\mathcal{P}ath\mathcal{S}hock$

continued from above.

	1990-2011		1990-2007	
n	$\mathcal{P}ath\mathcal{S}hock$	$\bar{R}^2$	$\mathcal{P}ath\mathcal{S}hock$	$\bar{R}^2$
	Specification 6			
2	0.35	11.64%	0.37	13.42%
	2.63		2.55	
3	0.34	11.29%	0.38	13.86%
	2.61		2.58	
4	0.33	10.82%	0.38	13.95%
	2.61		2.59	
5	0.30	8.84%	0.36	12.82%
	2.42		2.55	

Table XIX. Bond Return Predictability: $\mathcal{P}ath\mathcal{S}hock$ and g

The table reports the output from regressions of annual bond excess returns on a constant, expected monetary policy shocks, and levels of macroeconomic activity:

$$rx_{t+12}^{(n)} = const + \beta_{PS}^{(n)} \mathcal{P}ath\mathcal{S}hock + \beta_g^{(n)} g_t + \epsilon_{t+12}^{(n)}$$

Bond maturities (n) range from 2 to 5 years. Each panel reports the results for one of the 6 proxies of expected monetary policy described by Table I. The left panels report the results for the full sample (the last observation is the excess return that realized between 2011:7 and 2012:7), while the right panels report the results for the sample excluding the crisis (the last observation is the excess return that realized between 2007:6 and 2008:6). T-statistics, reported below the point estimates, are corrected for auto-correlation and heteroskedasticity using Newey-West errors (18 lags). $\bar{R}^2$ is the adjusted R^2. Both left and right hand variables are standardized. A constant is included but not reported.

	1990-2011			1990-2007		
n	$\mathcal{P}ath\mathcal{S}hock$	g	$\bar{R}^2$	$\mathcal{P}ath\mathcal{S}hock$	g	$\bar{R}^2$
			Specification 1			
2	0.37	-0.25	23.70%	0.25	-0.37	28.52%
	3.55	-2.14		2.08	-3.96	
3	0.36	-0.23	21.59%	0.27	-0.29	23.08%
	3.44	-2.23		2.17	-3.26	
4	0.36	-0.18	18.86%	0.29	-0.23	19.37%
	3.28	-1.72		2.21	-2.67	
5	0.33	-0.16	15.45%	0.29	-0.16	14.96%
	2.99	-1.78		2.25	-1.98	
			Specification 2			
2	0.35	-0.31	22.97%	0.23	-0.40	28.04%
	3.32	-2.70		1.85	-4.24	
3	0.35	-0.28	21.16%	0.25	-0.32	22.54%
	3.30	-2.89		1.95	-3.57	
4	0.36	-0.23	18.89%	0.26	-0.26	18.78%
	3.27	-2.41		2.00	-3.00	
5	0.33	-0.20	15.90%	0.27	-0.19	14.45%
	3.10	-2.50		2.08	-2.32	

Table XX. Bond Return Predictability: $\mathcal{P}ath\mathcal{S}hock$ and g

continued from above

	1990-2011			1990-2007		
n	$\mathcal{P}ath\mathcal{S}hock$	g	$\bar{R}^2$	$\mathcal{P}ath\mathcal{S}hock$	g	$\bar{R}^2$
			Specification 3			
2	0.33	-0.30	21.95%	0.25	-0.40	29.17%
	2.91	-2.13		1.95	-3.71	
3	0.33	-0.27	19.78%	0.28	-0.32	24.31%
	2.82	-2.23		2.09	-3.09	
4	0.32	-0.22	16.35%	0.31	-0.25	21.19%
	2.65	-1.82		2.22	-2.55	
5	0.29	-0.20	13.03%	0.32	-0.18	16.97%
	2.34	-1.87		2.31	-1.95	
			Specification 4			
2	0.35	-0.33	23.06%	0.25	-0.41	29.40%
	3.09	-2.51		1.97	-3.89	
3	0.34	-0.30	20.95%	0.28	-0.33	24.53%
	3.03	-2.65		2.10	-3.28	
4	0.34	-0.25	17.77%	0.31	-0.27	21.37%
	2.94	-2.22		2.22	-2.72	
5	0.31	-0.23	14.55%	0.32	-0.20	17.24%
	2.70	-2.28		2.31	-2.10	
			Specification 5			
2	0.31	-0.30	20.31%	0.22	-0.41	28.10%
	2.57	-2.05		1.69	-3.98	
3	0.30	-0.27	18.27%	0.26	-0.33	23.09%
	2.48	-2.14		1.83	-3.37	
4	0.29	-0.22	14.82%	0.28	-0.27	19.80%
	2.33	-1.75		1.95	-2.83	
5	0.26	-0.20	11.66%	0.29	-0.20	15.45%
	2.04	-1.79		2.01	-2.22	

Table XXI. Bond Return Predictability: $\mathcal{P}ath\mathcal{S}hock$ and g

continued from above

	1990-2011			1990-2007		
n	$\mathcal{P}ath\mathcal{S}hock$	g	$\bar{R}^2$	$\mathcal{P}ath\mathcal{S}hock$	g	$\bar{R}^2$
Specification 6						
2	0.33	-0.33	22.06%	0.23	-0.42	28.64%
	2.85	-2.43		1.78	-4.09	
3	0.33	-0.30	20.05%	0.27	-0.34	23.71%
	2.79	-2.56		1.91	-3.48	
4	0.32	-0.25	16.79%	0.29	-0.28	20.43%
	2.70	-2.14		2.02	-2.92	
5	0.29	-0.23	13.65%	0.30	-0.21	16.21%
	2.46	-2.20		2.10	-2.30	

Table XXII. Monetary Policy Shocks and Equity Returns

The table reports risk premium estimates (λ) for the 4-factor equity asset pricing model $E\left[RX^i\right] = \beta^{i\prime}\lambda$. The candidate risk factors are the market excess return (*mkt*), Fama and French (1993) value and size factors (*smb* and *hml*) and the portfolio mimicking monetary policy shocks (*mps*):

$$\beta^{i\prime} = \left[\beta^i_{mkt} \quad \beta^i_{smb} \quad \beta^i_{hml} \quad \beta^i_{mps}\right]'$$
$$\lambda' = [\lambda_{mkt} \quad \lambda_{smb} \quad \lambda_{hml} \quad \lambda_{mps}]'$$

Each specification uses *mps* constructed from one of the 6 proxies of expected monetary policy shocks described in Table I. Factor betas are estimated in first-stage time series regressions via OLS. For each specification: the first row reports (annualized) risk premia estimates; the second row reports t-statistics corrected for auto-correlation and heteroskedasticity using Newey-West errors (18 lags); the third row reports t-statistics that employ Shanken (1992) correction. Sample period: 1990:1 - 2011:7.

Specification	*mkt*	*smb*	*hml*	*mps*
1	6.17%	2.21%	4.03%	3.63%
	7.91	3.89	2.92	6.57
	1.83	0.83	1.60	1.45
2	5.99%	2.52%	3.79%	4.40%
	8.69	4.52	2.90	5.03
	1.77	0.95	1.50	2.17
3	6.43%	2.12%	4.16%	4.40%
	8.67	3.55	3.28	3.29
	1.91	0.80	1.65	2.42
4	6.41%	2.31%	3.89%	5.39%
	10.79	4.62	3.57	2.97
	1.89	0.87	1.54	2.68
5	6.27%	2.23%	4.13%	4.70%
	9.34	4.13	3.11	3.29
	1.86	0.84	1.64	2.53
6	6.41%	2.31%	3.89%	5.39%
	10.79	4.62	3.57	2.97
	1.89	0.87	1.54	2.68

Money Non-neutrality and Habit: an Example

This section presents a simple example of how money non neutrality and internal habit can interact to generate a link between monetary policy shocks and risk aversion. The intuition is simple. Internal habit implies that agents are concerned about the level of future consumption surplus which, under money non-neutrality, is sensitive to monetary policy shocks: since the local curvature of utility is inversely related to consumption surplus, monetary policy shocks have an impact on the coefficient of relative risk aversion and, therefore, on the price demanded by agents to bear a unit of risk.

A simple way to model money non-neutrality in a reduced form way is to allow lagged monetary policy shocks to affect the conditional mean of consumption growth. Let C_t and u_t denote the level of aggregate consumption and monetary policy shocks, respectively. Assume that the dynamics of $\Delta c_{t+1} = \log(C_{t+1}/C_t)$ and u_t are described by:

$$\Delta c_{t+1} = -\kappa u_t + \sigma_c \epsilon_t^c$$
$$u_{t+1} = \phi_u u_t + \sigma_u \epsilon_t^u.$$

The parameter κ controls the extent of money non-neutrality. If $\kappa = 0$ money neutrality holds: consumption growth is iid with mean zero and monetary policy has no effect on consumption. On the other hand, if $\kappa \neq 0$, money is non-neutral. Suppose, for instance, that the monetary cycle is contractionary ($u_t > 0$): the higher κ, the lower is the conditional mean of consumption growth. The AR(1) assumption for u_t reflects the mean-reverting nature of monetary policy shocks, and implies that the effect of monetary policy on the conditional growth of consumption is transitory: monetary policy cannot permanently affect the growth rate of the economy. Clearly, this process for consumption is only a simplistic representation; the process can be straightforwardly extended to include, for instance, a non-zero unconditional mean and additional non-monetary sources of variation in the conditional mean.

The representative agent maximizes

$$U_t(C_t, C_{t+1}, \ldots) = E_t \left[\sum_{j=0}^{\infty} e^{-\delta j} u_{t+j}(C_{t+j}, H_{t+j}) \right],$$

where $u_t(C_t, H_t)$ is a function that maps time-t consumption (C_t) and habit (H_t) into time-t utility. A common assumption in the literature (see, for instance, Campbell and Cochrane (1999), Wachter (2006), Buraschi and Jiltsov (2007), Santos and Veronesi (2010)) is that $u_t(C_t, H_t)$ is a CRRA utility function defined over the *difference* between consumption and habit:

$$u_{t+j}(C_{t+j}, H_{t+j}) = \frac{(C_{t+j} - H_{t+j})^{1-\gamma}}{1-\gamma}.$$

In general, habit is a function of current and past consumption:

$$H_t = f\left(C_t, C_{t-1}, C_{t-2}, ...\right);$$

typically, it is defined as an exponentially weighted average.

The main object of interest of this section is to understand how the coefficient of relative risk aversion

$$\eta_t = -C_t \frac{U_{cc}}{U_c},$$

varies as a function of monetary policy shocks. In general, the first and second derivative of utility with respect to consumption are given by:

$$U_c = \frac{\partial u_t\left(C_t, H_t\right)}{\partial C_t} + E_t\left[\sum_{j=1}^{\infty} e^{-\delta j} \frac{\partial u_{t+j}\left(C_{t+j}, H_{t+j}\right)}{\partial H_{t+j}} \frac{\partial H_{t+j}}{\partial C_t}\right]$$

$$U_{cc} = \frac{\partial^2 u_t\left(C_t, H_t\right)}{\partial C_t^2} + E_t\left[\sum_{j=1}^{\infty} e^{-\delta j} \frac{\partial\left(\frac{\partial u_{t+j}\left(C_{t+j}, H_{t+j}\right)}{\partial H_{t+j}} \frac{\partial H_{t+j}}{\partial C_t}\right)}{\partial C_t}\right].$$

With external habit, agents ignore the impact of today's consumption on future habits ($\partial H_{t+j}/\partial C_t = 0$), so the expectation terms are zero and the first and second partial derivatives simplify to $\partial u_t\left(C_t, H_t\right)/\partial C_t$ and $\partial^2 u_t\left(C_t, H_t\right)/\partial C_t^2$, respectively. Since monetary policy affects the real economy (consumption) only with a lag (u_t affects C_{t+j} but not C_t), it is clear that RRA is independent of monetary policy shocks when when habit is external. Under internal habit, on the other hand, agents do not ignore the impact that current consumption has on future habit ($\partial H_{t+j}/\partial C_t \neq 0$), so that U_c and U_{cc} retain the general form illustrated above and RRA depends on u_t via C_{t+j}.

In order to illustrate the relationship between u_t and η_t, assume that utility is defined over the ratio of consumption to habit and that habit is a (linear) function of lagged consumption:

$$U_t\left(C_t, C_{t+1}, \ldots\right) = E_t\left[\sum_{j=0}^{\infty} e^{-\delta j} \frac{\left(C_{t+j}/H_{t+j}\right)^{1-\gamma}}{1-\gamma}\right]$$

$$H_t = hC_{t-1}.$$

Modeling the wedge between consumption and habit as a *ratio* implies a constant coefficient of risk aversion under external habit but not under internal habit.[23] We choose the specification where utility is a function of C_t/H_t, instead of $C_t - H_t$, because of analytical tractability (we can compute the expectation terms in the former case). The specification for the habit function is highly unrealistic, but it serves the purpose to simplify calculations: $\partial^n H_{t+j}/\partial C_t^n \neq 0$ only for $j = 1$ and $n = 1$, so the impact of current consumption on future habits is limited to first order

[23]The former result partly explains the assumption, common in external habit models, that utility is a function of the *difference* between consumption and habit.

effects and to the first horizon. Under these assumptions the expression for RRA can be computed in closed form. The first and second derivative of utility are:

$$
\begin{aligned}
U'_C &= (C_t/H_t)^{-\gamma} H_t^{-1} - E_t\left[\sum_{j=0}^{\infty} e^{-\delta j} C_{t+j}^{1-\gamma} H_{t+j}^{-(2-\gamma)} \frac{\partial H_{t+j}}{\partial C_t}\right] \\
&= (C_t/H_t)^{-\gamma} H_t^{-1} - E_t\left[e^{-\delta} C_{t+1}^{1-\gamma} H_{t+1}^{-(2-\gamma)} h\right] \\
&= (C_t/Ht)^{-\gamma} H_t^{-1} - E_t\left[e^{-\delta} C_{t+1}^{1-\gamma} (hC_t)^{-(2-\gamma)} h\right] \\
&= (C_t/H_t)^{-\gamma} H_t^{-1} - e^{-\delta}(hC_t)^{-(2-\gamma)} h E_t\left[C_{t+1}^{1-\gamma}\right] \\
&= (C_t/H_t)^{-\gamma} H_t^{-1} - e^{-\delta}(hC_t)^{-(2-\gamma)} h C_t^{(1-\gamma)} e^{-(1-\gamma)\kappa u_t + 0.5(1-\gamma)^2\sigma^2} \\
&= (C_t/H_t)^{-\gamma} H_t^{-1} - e^{-\delta} h^{\gamma-1} C_t^{-1} e^{-(1-\gamma)\kappa u_t + 0.5(1-\gamma)^2\sigma^2} \\
U''_{CC} &= -\gamma (C_t/H_t)^{-\gamma-1} H_t^{-2} \\
&\quad - E_t\left[\sum_{j=0}^{\infty} e^{-\delta j} C_{t+j}^{1-\gamma}\left(-(2-\gamma) H_{t+j}^{-(3-\gamma)}\left(\frac{\partial H_{t+j}}{\partial C_t}\right)^2 + H_{t+j}^{-(2-\gamma)} \frac{\partial^2 H_{t+j}}{\partial C_t^2}\right)\right] \\
&= -\gamma (C_t/H_t)^{-\gamma-1} H_t^{-2} - E_t\left[e^{-\delta} C_{t+1}^{1-\gamma}\left(-(2-\gamma) H_{t+1}^{-(3-\gamma)}\left(\frac{\partial H_{t+1}}{\partial C_t}\right)^2\right)\right] \\
&= -\gamma (C_t/H_t)^{-\gamma-1} H_t^{-2} - E_t\left[e^{-\delta} C_{t+1}^{1-\gamma}\left(-(2-\gamma)(hC_t)^{-(3-\gamma)} h^2\right)\right] \\
&= -\gamma (C_t/H_t)^{-\gamma-1} H_t^{-2} + (2-\gamma) e^{-\delta}(hC_t)^{-(3-\gamma)} h^2 E_t\left[C_{t+1}^{1-\gamma}\right] \\
&= -\gamma (C_t/H_t)^{-\gamma-1} H_t^{-2} + (2-\gamma) e^{-\delta}(hC_t)^{-(3-\gamma)} h^2 C_t^{(1-\gamma)} e^{-(1-\gamma)\kappa u_t + 0.5(1-\gamma)^2\sigma^2} \\
&= -\gamma (C_t/H_t)^{-\gamma-1} H_t^{-2} + (2-\gamma) e^{-\delta} h^{\gamma-1} C_t^{-2} e^{-(1-\gamma)\kappa u_t + 0.5(1-\gamma)^2\sigma^2},
\end{aligned}
$$

where we have used the fact:

$$
E_t\left[C_{t+1}^{\theta}\right] = C_t^{\theta} e^{-\theta\kappa u_t + 0.5\theta^2\sigma_c^2},
$$

which follows directly from the dynamics for C_t and the assumption of lognormality. It follows that the RRA is equal to:

$$
\eta_t = C_t \frac{\gamma (C_t/H_t)^{-\gamma-1} H_t^{-2} + (\gamma-2) e^{-\delta} h^{\gamma-1} C_t^{-2} e^{-(1-\gamma)\kappa u_t + 0.5(1-\gamma)^2\sigma^2}}{(C_t/H_t)^{-\gamma} H_t^{-1} - e^{-\delta} h^{\gamma-1} C_t^{-1} e^{-(1-\gamma)\kappa u_t + 0.5(1-\gamma)^2\sigma^2}}
$$

Figure 11 shows the coefficient of RRA as a function of monetary policy shocks. The plot is based on the following assumptions: (i) CRRA curvature: $\gamma = 5$; (ii) time preference: $\delta = 0$; (iii) non-neutrality parameter: $\kappa = 4$ (a 25 basis points monetary policy shock reduces $E_t[\Delta c_{t+1}]$ by 1%); (iv) initial consumption and habit: $C_t = H_t = 1$ (all the variation in RRA stems from expectation terms); (v) habit parameter $h = 0.75$ (low enough to ensure that consumption does not fall below habit); (vi) consumption volatility: $\sigma_c = 1\%$. The magnitude of monetary policy shocks encompasses the range of potential rate cuts/increases: $u_t = 0, \pm 25bp, \pm 50bp, \pm 75bp$. The figure shows that monetary policy shocks have a non trivial effect on relative risk aversion: the price of risk is an increasing function of u_t.

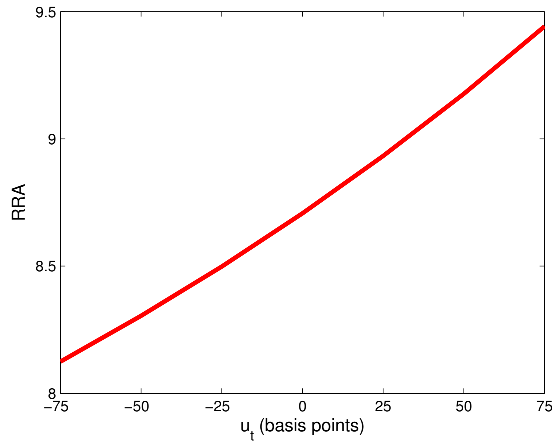

Figure 11. Monetary policy shocks and RRA

This figure plots the coefficient of relative risk aversion RRA as a function of monetary policy shocks u_t for a calibrated economy with internal habit and money non neutrality discussed in the Appendix. The plot is based on the following assumptions: (i) CRRA curvature: $\gamma = 5$; (ii) time preference: $\delta = 0$; (iii) non-neutrality parameter: $\kappa = 4$ (a 25 basis points monetary policy shock reduces $E_t[\Delta c_{t+1}]$ by 1%); (iv) initial consumption and habit: $C_t = H_t = 1$, so that all the variation in RRA stems from expectation terms); (v) habit parameter $h = 0.75$; (vi) consumption volatility: $\sigma_c = 1\%$. The magnitude of monetary policy shocks encompasses the range of potential rate cuts/increases: $u_t = 0, \pm 25bp, \pm 50bp, \pm 75bp$.