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Liquidity risk and specialness ☆

Andrea Buraschi[a,*], Davide Menini[a,b]

[a] London Business School, Institute of Finance, Sussex Place, Regents Park, London NW1 4SA, UK
[b] Morgan Stanley, UK

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Abstract

Repo contracts, the most important form of collateralized lending, are widely used by financial institutions and hedge funds to create short-selling positions and manage their leverage profile. Moreover, they have become the primary tool of money management and monetary control of several central banks, including the Bundesbank and the newly born European Central Bank. This paper is an empirical study of this market. More specifically, we study the extent to which the current term structure of long term "special" repo spreads discount the future collateral value (specialness) of Treasuries. We ask whether repo spreads embed a liquidity risk premium and whether such a risk premium is time-varying. We quantify the size of the average liquidity risk premium and we provide empirical evidence of the extent of its time-variation. © 2002 Elsevier Science B.V. All rights reserved.

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*Corresponding author. Tel.: +44-207-431-3771.
E-mail address: aburaschi@london.edu (A. Buraschi).

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1. Introduction

The collateral value[1] of Treasury bonds can vary quite substantially across different issues. Squeezes, auction cycles, deliverability against future contracts, short covers, and cornerings can make the collateral value of specific Treasury issues vastly different from the collateral value of a "general" Treasury bond.

Repo rates reflects the collateral and liquidity value of traded securities as the repo market is the most important form of collateralized lending. The extremely fast development of a very active market in derivatives has been paralleled by an increased demand for instruments that can facilitate the implementation of dynamic hedging strategies. Repos are widely used by financial institutions and hedge funds to create and manage their risk profile. Although the repo market is one of the most important markets by size, it has been overlooked by the empirical literature. In this paper, we would like to explore some empirical regularities of this market. In particular, we ask the following two questions: (a) to what extent do long term-repo spreads anticipate and price the future relative scarcity value of bonds? and (b) is liquidity risk priced by the term structure of repo spreads and, if this is indeed the case, is the liquidity risk premium also time-varying?

Longstaff (2000b) finds that the expectation hypothesis is not rejected for extremely short rates (up to three months). These results contrast with the evidence on long term yields and shed new light on the behavior of fixed income markets. Longstaff interprets his finding saying, "Our results support the widespread view that much of the apparent term premium in Treasury Bills is actually due to other factors such as liquidity". His findings are very important and in this study we focus directly on the empirical behavior of what is widely considered in Wall Street to be a good proxy for liquidity: the repo spread. This study is made possible by a unique data set that contains daily data on term-repo rates on *all the individual* German Government bonds which traded special, with tenors up to three months. This allows us to construct a term structure of repo spreads and explore the extent to which forward repo spreads anticipate the future specialness of Treasury bonds.

Duffie (1996) discusses the relationship between the value of a bond in the repo market and in the cash market. He shows that a bond that trades "special" in the repo market should trade at a price premium in the cash market. The size of the cash premium should be a function of two factors: (1) the extent of future specialness measured by the size of the future repo spread and (2) the length of time during which the bond will trade "special". Jordan and Jordan (1997) regress the special bond price premium on the overnight "specialness" and find strong support for the fact that the overnight specialness in the repo market is reflected in a cash premium in the spot market.

[1] We say that two bonds have different collateral value if the cost of borrowing in a collateralized loan contract differs depending on the type of bond offered as collateral. The bond specific financing cost is referred to as the "special" repo rate; the difference between the "general collateral" and the "special" repo rate is defined as the "repo spread" and it is a particular form of convenience yield.

In this paper, we consider a related, but different, issue. If the market efficiently anticipates that some bonds will be financed at lower repo rates on a rolling basis for some time in the future, then it should require an adequate *term*-repo spread on that issue. What is the empirical evidence that the repo market correctly prices the total future convenience yield generated by bonds on special? We use a simple two-factor model to show that this question can be addressed in the general framework of the expectations hypothesis.

Several economic reasons motivate the interest in empirical properties of the term structure of special repo spreads:

(1) Collateral value differs greatly between traded instruments. Compare for example repo rates of bonds that are on special with the repo rates on general collateral bonds. Sometimes this spread can be as high as one hundred basis points even in low interest rates environments and even for largely traded Treasury securities. Yet, most value-at-risk (VaR) measures of risk simply ignore these differences. Thus, when these measures are used to adjust trading profits, using the risk adjusted return on capital (RAROC), in order to compare the performance of business units, they bias the performance measures in favor of trading desks which invest in low collateral value instruments. The issue about which measure to use for the cost of capital that could take into account not just the market risk, as the VaR approach does, but also some proxy of collateral value has never been more debated than in the aftermath of the 1998 financial turmoil.

(2) In the pricing and marking-to-market of options and other bond derivatives, the knowledge of the term structure of repo spreads is necessary to compute the forward price of the underlying bond. This is even more important since options and other derivatives contracts are usually written on the same benchmark securities that more frequently go on special. An important example of a claim that is very sensitive to relative costs of carry of different bonds is the quality option embedded in virtually all bond futures.

If the current term structure of repo spreads were to anticipate correctly (on average) the future scarcity value of financial assets, it has been argued that one could form a proxy for the collateral value using the current term structure of repo spreads: a simple, observable and forward looking market-based measure. Other proxies that have been suggested include current bid–ask spreads and trading volumes. However, these proxies have the limitations of not being forward looking. What is the empirical evidence on this issue?

Although specialness is clearly accounted for in the cash market, as found in Jordan and Jordan (1997), we find strong violations of the expectations hypothesis: current forward spreads overestimate changes in future specialness.

We test the robustness and power of the econometric approach using Monte Carlo methods and discuss the reasons for such a violation. We explore if the violation of the expectation hypothesis is due to the existence of a time-varying risk premium and estimate a Garch-in-mean to test whether the time-varying conditional volatility of the repo spread is a priced risk factor.

This paper is organized as follows. In the remainder of this section we give a brief account of the institutional details of the repo market. In Section 2 we describe the testable restrictions that will be the object of our analysis. In Section 3 we present our data set. Section 4 contains our econometric tests, and Section 5 uses Monte Carlo experiments to assess the robustness of our results to small sample biases and measurement error. In Section 6 we discuss whether deviations from the expectation hypothesis can be due to the existence of a time-varying risk premium and we study the economic pattern of this risk premium. Finally, Section 7 suggests an economic interpretation of the results and concludes.

1.1. The repo market

A repo is a single transaction that combines a spot market sale of a security and a simultaneous forward agreement to repurchase the same security at a later date at a price that reflects a financing cost equal to the repo rate. Duffie (1996) provides a detailed description of the repo interest rate calculation. If the repurchase agreement is to be completed the following day, the contract is defined as an overnight repo. If the delivery of the security is to take place at a later date, it is referred to as a term repo contract.

We say that an investor has "done a repo" if the first leg of the transaction involves the sale of a security, so that the cash balances of the investor at the initiation of the contract have increased. The counterpart in this transaction is said to have done a "reverse repo" and he or she is bound to deliver the security at the maturity of the repo contract.

Treasury bond repo markets are generally very active, competitive, and liquid as they are used to achieve several objectives:

1. *Collateralized lending*. The repo market can be thought of as the market for collateralized lending since a repo contract effectively takes the form of a collateralized loan. When an investor enters into a reverse repo, he is lending to the counterpart while keeping the underlying security as collateral. The collateralized lending rate is the repo rate. The element of collateralization in the loan agreement reduces the cost of default and therefore the cost of capital.

2. *Short-selling*. Reverse repo contracts are often used to create short positions in Government or Corporate bonds. An investor would sell spot the security obtained as collateral in the reverse repo contract. At the expiration of the contract, the investor will cover his obligations by purchasing the underlying asset in the cash market and delivering it to the counterpart. Given its high level of liquidity and small transaction costs, the repo market is often the preferred vehicle to create short-selling positions. The fast growth of the derivatives market, and the necessity of implementing dynamic hedging strategies, has been a major catalyst in the development of the repo market.

3. *Leverage*. If an investor owns a security, he can increase his leverage by doing a repo and using the proceeds to increase the exposure. A celebrated example was Orange County. The value of the assets managed by Robert Citron were equal to

$7.5 billion, however the positions were for $20 billion, almost entirely financed on the repo market. The development of efficient repo markets on sovereign debt, corporate bonds, equity, and mortgage-backed securities has played an important role in the increase of the leverage capacity of institutional operators and hedge funds. The financial turmoil that characterized the summer of 1998 revealed the extent of leverage that some hedge funds have been able to create using the repo market. Repo transactions also imply the use of credit lines and have non-zero capital requirements costs,[2] thus limiting the desired level of risk exposure.

4. *Monetary policy*. Repos have become the most important tool of monetary policy of several Central Banks, including Bundesbank.[3] Choudhry (1999) claims that the "[...] Bundesbank actively uses repo in all aspects of its money market operations. This includes long term adjustments that involve outright bond purchases, interim financing using repo and "fine tuning" using two to ten days repo". Moreover, the European Monetary Institute has intimated that its successor, the European Central Bank in Frankfurt, will use the repo market for open market operations after the introduction of the euro.[4] As a consequence, repo rates are becoming a money market benchmark in some of the world largest fixed income markets.

1.2. Special repo rates

Since a repo contract is specific to the asset that serves as collateral, the repo rate may vary depending on the current and expected future demand of this asset. At any given date, most issues are perceived to have the same relative collateral value. In this case they trade at the same repo rate, called the *general collateral* rate.

However, some issues may trade at a lower repo rate, indicating that the convenience yield of the bond is high. These assets are said to be *on special* and the collateralized lending rate is called a *special repo rate*. If an investor owns securities that go on special, he can finance his inventories at a lower cost. However, it is also the case that the relative cost of short-selling securities that are on special is higher than for securities that trade at general collateral. The repo spread is defined as the difference between the general collateral repo rate and the special repo rate.

Duffie (1996) proposes an equilibrium model that shows how demand and supply considerations can influence repo spreads of bonds on special. Moreover, he discusses the link between repo spreads and the relative value, in the cash market, of

[2] The BIS makes a distinction between "banking book" transactions as carried out by retail and commercial banks (primarily deposits and lending) and "trading book" transactions as carried out by investment banks and security houses. A repo transaction carries an 8% "charge" on the trading book on the book-to-market net exposure of the transaction multiplied by the risk weighting of the counterpart.

[3] Nautz (1997) claims that "The use of repos in the provision of central bank money have increased dramatically in the second half of the 1980s. As a consequence, the repo rate governs the term structure of interest rates in the money market while the Bundesbank's "key interest rates" (that is, discount and Lombard rate) have lost much of their traditional significance".

[4] It is estimated that the repo market accounts for up to 50% of daily settlement activity in non-U.S. Government bonds world-wide.

bonds on special with respect to bonds trading at general collateral. The higher collateral value of a bond on special can be interpreted as a convenience yield that is reflected in a higher spot price of the security in the cash market. He also discusses the reasons why certain Treasury bonds become special.

Reasons that can drive a Treasury bond to trade special include the tightness of the auction when the security is first issued. When the auction has a high bid-to-cover ratio, a large number of unsuccessful bidders need, after the auction, to cover the short positions usually accumulated in "when-issued" trading. In support of this conjecture—and in contradiction to the winner's curse—Jagadeesh (1993) finds that agents submitting winning bids in particularly tight auctions tend to realise higher than average profits. Duffie (1997) notes that shorts in the "when-issued" markets are likely to cover their position through reverse repo, thus contributing to making the auctioned bond special.

Using the same line of argument, a large relative volume of inventory held "off-the-street" may lead the newly auctioned bond to trade special. Jordan and Jordan (1997) find supporting evidence for this effect.

Many Treasury bonds trade special well away from their auction cycle. Some of the reasons may include the deliverability against the futures and the benchmark status of the bond. Squeezes in the future and cash markets, asset swap flows, swings in portfolio allocations due to large market shocks, and other liquidity shocks can lead a bond to trade at large repo spreads. On occasion, the spreads may be so wide that the Treasury can decide to reopen the issue and provide the market with additional bonds with identical characteristics or to intervene in the secondary market with market operations.

2. The empirical testable restrictions

We now turn to explore the link between the expectation hypothesis for interest rates and for repo spreads. We focus on a model in which the value of a bond depends on the cash flows, coupons plus principal, and its collateral value.[5] We follow Duffie (1996) and assume that the price process of bonds that are not trading special depends on one stochastic factor that is naturally interpreted as (general collateral) interest rate risk. Let us define r_t to be the general collateral instantaneous rate. In our framework, the instantaneous general collateral rate plays the same role as the instantaneous interest rate in traditional term structure models. This can be justified by the minimum level of credit risk implied by the quality of the collateral in a Government bond repo transaction. It is both the rate at which (1) holdings in a

[5] Duffie and Singleton (1997) discuss bond pricing idiosyncracies that can arise from liquidity, credit risk, or fiscal reason in the general framework of term structure models. Fisher and Gilles (1996) study the repo market in the framework of a Heath et al. (1992) model and describe the no-arbitrage restrictions that need to be satisfied by bonds on special. They show that, for these bonds, the classical HJM restrictions on the drift of the forward rate process needs to be corrected by an additional term that is a function of the instantaneous covariance between the forward spread and the return on a general collateral bond. Cherian et al. (1999) propose a pricing model for bond with convenience yield in a HJM framework.

non-special bond can be financed in the repo market and (2) the instantaneous rate of growth of the money market account ($\exp \int_0^t r_s \, ds$) used as a numéraire to deflate asset prices under the associated equivalent martingale measure.

Prices of bonds trading special,[6] on the other hand, are affected both by pervasive shocks to the level of general collateral interest rates, and by security specific shocks to repo spreads. Let the bond specific special repo rates process be s_t^i. This can be written as the difference between the general collateral rate and the bond specific repo spread ℓ_t^i. The choice of this notation is consistent with the literature, Grinblatt (1994) and Duffie and Singleton (1997), which tends to identify the repo spread ℓ_t^i as a proxy for liquidity. Let us define $P_s^i(t, \tau)$ as the price at time t of a special zero coupon bond maturing at τ. Under some technical conditions, Duffie (1996) shows that the existence of no-arbitrage opportunities implies that (1) $\ell_t^i \geqslant 0$ and, assuming that the repo spread captures all idiosyncracies related to the bond, such as for instance liquidity differential, preferential tax treatment, and credit quality, (2) under the equivalent martingale measure dQ, the instantaneous expected return of bonds on special is such that $E_t^Q[dP_s^i(t, \tau)/P_s^i(t, \tau)] = r_t - \ell_t^i$.

Condition (1) states that the existence of a negative repo spread generates an arbitrage opportunity, similarly to the case of negative interest rate in the presence of a zero cost technology to store money.

Condition (2) clarifies the role of the repo spreads as a particular convenience yield. This diminishes the cost of financing a position in a bond trading special and therefore it lowers the risk neutral expected rate of appreciation. Benchmark bonds and bonds that are deliverable against the futures are both more liquid and likely to be on special. Moreover, Duffie shows that of two otherwise identical securities the most liquid is also the most likely to be on special.

Within this framework, the no-arbitrage price time t of a zero coupon special bond maturing at time τ can be written as

$$P_s^i(t, \tau) = E_t^Q\left[\exp - \int_t^\tau (r_s - \ell_s^i) \, ds\right], \tag{1}$$

where E_t^Q denotes conditional expectations taken with respect to the risk neutral equivalent martingale measure. If the bond specific special repo spread is idiosyncratic (uncorrelated) to r_s, it is possible to express the cash premium as

$$\frac{P_s^i(t, \tau)}{P(t, \tau)} = E_t^Q\left[\exp \int_t^\tau \ell_s^i \, ds\right] \tag{2}$$

$$= \exp[L^i(t, \tau)(\tau - t)]. \tag{3}$$

The cash premium is equal to the risk neutral conditional expected value of the future repo advantages measured over the life of the bond, that in turn can be locked

[6] We are considering the case of zero coupon bond prices here, while in general special bonds are coupon bearing. Clearly, the pricing restrictions hold as an approximation for coupon bearing bonds. However, Fisher and Gilles (1996) argue that the approximation error is immaterial for short term repo transactions.

in by entering into a term-repo transaction at a spread of $L^i(t,\tau)$ up to bond maturity. This condition is recognizable as the repo market analog of the well known bond pricing relationship that serves as a basis for several tests on the expectations hypothesis.

The expected convenience yield of bonds currently trading special depends on two elements. First, it depends on the expected future level of specialness, measured ex post by the future short-term repo spread ℓ^i_s. Second, it depends on the expected length of time the bond remains traded at a special collateral repo rate. What is the evidence on the extent to which current long-term tenor repo spreads $L^i(t,\tau)$ (of bonds currently on the special) price correctly future convenience yields?

In order to address this question, we consider the standard framework of the expectations hypothesis of the term structure of interest rates (henceforth EH). The EH of the term structure states that agents' expectations about realization of future interest rates are fully embodied in the present level of interest rates, and therefore bond prices. In its pure form, i.e., without any allowance for the existence of a risk premium, it is alternatively formulated saying that (1) forward rates are unbiased predictors of futures rates (unbiased expectations hypothesis: U-EH), (2) the instantaneous expected return on bonds of any maturity is equal to the instantaneous interest rate (local expectations hypothesis: L-EH), or (3) the expected return from rolling over a money market account up to a certain maturity is equal to the return that can be obtained by purchasing an equal maturity zero coupon bond (returns to maturity expectations hypothesis: RTM-EH).[7]

Traditional tests of the EH are usually based on a version of the U-EH. Although Cox et al. (1981) claim that this specification is incompatible with any continuous time rational–expectation economy, McCulloch (1993) shows an example of a stochastic endowment economy that supports bond prices satisfying the U-EH in its pure form (no term premium). Longstaff (2000a) discusses in detail examples of economies in which the EH is consistent with the hypothesis of no-arbitrage. In this paper, we explore the extent to which long term special repo spreads discount future bond specialness in the framework of a generalized U-EH that includes a (constant) risk premium.

When the expectation hypothesis is tested on the level of long-term US yields, the null hypothesis is usually rejected. However, Longstaff (2000a, b) finds that the expectation hypothesis is not rejected when tested on the level of very short-term interest rates (up to three months), the same tenors that are used in our study. His findings make the expectation hypothesis a reasonable model-independent framework to study the empirical issue of the existence of a time-varying liquidity premium.

[7] There is a fourth version of the expectation hypothesis, namely the yield to maturity expectation hypothesis (YTM-EH) stating that the expected yield from rolling over instantaneously a money market account up to a certain maturity is equal to the one obtained by purchasing an equal maturity zero coupon. It can be easily shown that this formulation is exactly equivalent to the Unbiased Expectation Hypothesis, as discussed in Cox et al. (1981). Campbell (1986) shows how the expectation hypothesis can be obtained as a log-linear approximation of the bond price process.

Even at longer horizons, one may argue that the expectation hypothesis is an interesting model-free framework of analysis. Repo spreads are bond specific variables and their innovations may be related to purely idiosyncratic (collateral specific) liquidity shocks. The correlation between the general level of the interest rates and the special repo spread is very small. Thus, ex ante, there is no reason to expect a rejection of the EH on repo spreads. If this were the case it would be due to other economic reasons.

The U-EH with constant risk premium can be formalized as

$$\pi_{\text{gc}} = f_{\text{gc}}(t,u) - \mathrm{E}_t^P[r_u], \tag{4}$$

$$\pi_{\text{s}}^i = f_{\text{s}}^i(t,u) - \mathrm{E}_t^P[r_u - \ell_u^i] \tag{5}$$

for the general collateral and the bond on special respectively. We define $f(t,u)$ as the instantaneous forward rate that satisfies the pricing relation $P(t,\tau) = \exp(-\int_t^\tau f(t,u)\,\mathrm{d}u)$. Let us consider the spread on the risk premia and forward rates $\pi^i = \pi_{\text{gc}} - \pi_{\text{s}}^i$ and $f^i = f_{\text{gc}} - f_{\text{s}}^i$ respectively, so that

$$\pi^i = f^i(t,u) - \mathrm{E}_t^P[\ell_{\text{s}}^i(u)] \tag{6}$$

which gives an U-EH restriction for the repo spread. To avoid confusion, in what follows we will simply refer to the "spread expectation hypothesis" with the understanding that we refer to the restriction in Eq. (6).

Integrating Eq. (6), we can obtain that

$$L^i(t,T) = \frac{\int_t^T (\mathrm{E}_t^P(\ell_{\text{s}}^i(u) + \pi^i)\,\mathrm{d}u}{T-t}. \tag{7}$$

A direct discretization of Eq. (7) gives a testable restriction that has been extensively studied in empirical studies on the term structure of interest rates:

$$L^i(t,t+n\Delta t) = \frac{\sum_{i=1}^n \mathrm{E}_t^P[L^i(t+(i-1)\Delta t, t+i\Delta t)] + \pi^i}{n\Delta t}. \tag{8}$$

Eq. (8) states that the n period term-repo spread is a simple average of a series of one period expectations of future repo spreads, plus a constant risk premium π^i.

Within this context, if the risk premium is constant, changes in the term-repo spreads can only be explained by changes in the expectation of the future level of repo spreads. On the other hand, if the risk premium is time-varying, spot rate changes can also depend on changes of investors' attitudes towards risk or changes in repo spread volatility.

Since common statistical tests generally find high persistence in interest rates time series, as documented in Anderson et al. (1997), in order to reduce the possibility of generating spurious results, most empirical tests on the expectations hypothesis focus on interest rate changes, rather than levels. From Eq. (8), it is possible to derive (see Campbell and Shiller, 1991) the following two testable restrictions:

type A restriction (long-term):

$$\mathrm{E}_t[L^i(t+\Delta t,T)] - L^i(t,T) = \alpha^i + \beta^i\left[\frac{\Delta t}{T-(t+\Delta t)}\right][L^i(t,T) - L^i(t,t+\Delta t)], \tag{9}$$

type B restriction (short-term):

$$\sum_{j=1}^{n-1}\left(1-\frac{j}{n}\right)\mathrm{E}_t(\Delta^j L^i) = \alpha^i + \beta^i[L^i(t,T) - L^i(t,t+\Delta t)], \tag{10}$$

where $\Delta t = (T-t)/n$, $\Delta^j L = L(t+i\Delta t, t+(i+1)\Delta t) - L(t+(i-1)\Delta t, t+i\Delta t)$ and the index i $(i = 1, \dots, 12)$ denotes different special bonds.

Eq. (9) relates the slope of the term structure of repo spreads (proxied by the difference between T and $t+\Delta t$ rates) to future changes in the T (i.e., the long-term) repo spread after a correction for a term premium. On the other hand, Eq. (10) relates the slope of the term structure to future changes in the short-term interest rates.

In both cases, if the U-EH is satisfied, the value of the β coefficient is equal to one, whereas α accounts for the risk premium and the Jensen's inequality effect. Note that a positive risk premium translates into negative values for α. On the contrary, the impact of the Jensen's effect[8] translates into positive values of α. Abstracting from convexity effects, if the spread EH held in its *pure* form, we should expect a zero intercept for both equations.

It can be noticed that if repo spreads' squeezes were idiosyncratic, the intercepts of Eqs. (9) and (10) would be equal to zero, even if the risk premium on general collateral were strictly positive. Since the factor ℓ_t^i captures idiosyncrasies that are specific to bond i that is trading special, it may be argued that these shocks are more diversifiable than shocks on the general level of the interest rates. A corollary of this hypothesis is that repo spread risk is diversifiable and does not carry a risk premium. We discuss the relevance of this assumption in the empirical section of the paper. If this conjecture were correct, then the EH on the term structure of repo spread would be satisfied and it could be used as a proxy for the future scarcity value of bonds. In the next sections we study the empirical evidence on this hypothesis.

3. The data set

Our study is based on the German Government bond repo market. This market is one of the oldest, most developed, and liquid repo markets, as well as the most important in Europe. The UK repo market is younger and generally considered less liquid than the German one. In 1998, the estimated daily trading volume in the domestic German repo market has been U.S. \$250 billion, little short of the U.S. Treasury repo market, with around 35 houses actively trading contracts. The profile and acceptance of repo transactions has been helped by the Bundesbank's use of the repo market as a tool for implementing monetary policy. Moreover, the European Central Bank in Frankfurt will use the repo market for open market operations to control the monetary base, after the introduction of the euro. This, and the

[8] Empirical analysis shows that the convexity bias is completely immaterial given the short-term nature of the contract that we are studying (up to three months).

leadership role played by Germany, is establishing the German Treasury repo rate as the interest rate benchmark of the new European financial environment.

The data set on special repo rates contains daily quotes (closing rates) on all repo contracts, including both general collateral and special, that are available on each individual Treasury bond from March 12, 1996 to October 6, 1998. Although the German repo market is extremely liquid and competitive (bid–ask spreads range between two and four basis points), as for the U.S. Treasury market all transactions are over-the-counter. The data had to be collected directly from dealers and the clearing house. No commercial provider records and supplies special *term* repo rate data on U.S. Treasury bonds and/or German Bunds.

Each daily observation consists of quotes on general collateral repo rates and special repo rates for four different tenors, respectively one week, one month, two months and three months. Repo quotes are not transaction data but represent the prices at which the trader, on market close, is ready to initiate a trade. No asynchronous trading problem is therefore present.

The number of German bonds that went special is a large fraction of the total government bond market. Over the period considered, the number of bonds trading special ranges from 19 to 33, out of a total of 81 to 91 Government bonds. In the large majority of cases, bonds that trade special on the repo market are those deliverable against the futures in the three different contracts (two, five and ten year maturity). Some exceptions include long term bonds (i.e., 15 and 30 year bonds). We observe rare cases of special bonds of intermediate maturities which are nondeliverable.

The percentage of bonds "on the special" in the German market appears higher when compared to the U.S. market. As an example, Sundaresan (1994) documents a ratio of 5–10% in his analysis of U.S. Treasury auctions from 1980 to 1991. Jordan and Jordan (1997) consider a sample ranging from September 1991 to December 1992 and find that, in this period, an average of four to six notes out of approximately 130 outstanding issues show some degree of specialness. This difference can be due to the substantially higher active role played by the U.S. Treasury in reopening Treasury issues that goes on special. This active role became very evident especially after the Solomon squeeze scandal. An additional reason is the higher degree of variability in the status of cheapest-to-deliver in the German market, especially in the five year segment.

Furthermore, our data set is unique in that it permits the estimation, at any point in time and for any special bond, of the entire term structure of repo spreads of different tenors. Therefore, it permits a rigorous empirical investigation of the extent to which current repo spreads price correctly future convenience yields generated by bonds trading special as suggested by the theory. The data set allows us to investigate this question both by considering a large cross-section of *individual* bonds, and also by considering different *investment horizons* (up to three months) since we can observe contracts of different tenor.

The analysis in Jordan and Jordan (1997) is based on a data-set that includes only overnight repo spreads, not individual term-repo spreads, and it focuses on a different issue, namely the relation between the cash premium and the repo spread.

They find evidence that (1) overnight repo specialness is reflected in a higher cash value of the bond, (2) on-the-run bonds trade rich with respect to off-the-run issues, and (3) the cash premium is influenced by auction tightness and the percentage allocated to dealers.

The number of bonds that have traded special at least one day has been 66. For these bonds, the average number of days on the special is 138. In order to have a sufficiently long sample size to run regression analysis, we first filter these bonds to consider the ones that have traded special at least 100 days. The number of these bonds is 43. Among these bonds the ones that have exhibited a special repo spread of at least 100 basis points are 12. Let $L^i(t, t+1w)$ be the one week repo spread. Spreads have been converted to a continuously compounded basis. The notation $1d, 1w$, and $1m$ denote one day, one week, and one month periods expressed as fractions of a year. Table 1 shows the number of German government bonds that have traded at a repo spread larger than a given threshold and that traded special for a given number of days.

Comparing the results in Table 1 with the evidence reported by Jordan and Jordan (1997), in the U.S. Treasury market the average number of days bonds trade (overnight) "on special" is substantially smaller and equal to 28.5. Also in this sense, the German market is a rich laboratory to study empirical regularities in the special repo market.

Table 2 reports some descriptive statistics on the 12 bonds included in the restricted dataset. Some interesting points emerge. First, we find that bonds can remain special for a considerable amount of time. The average number of trading days that these 12 bonds traded special is 195, that compares with an unconditional average of 138 days for the all sample. If we consider bonds that reached maximum repo spreads of 50 basis points (bp), we find that 15 bonds traded special in the sample period for at least 150 days and eight bonds traded special for at least 200 days (almost 60% of the sample period).

Second, repo spreads tend to be small in absolute values but quite volatile. The median repo spread value is about $23bp$, over the entire sample, with a standard deviation of $13bp$. The results also show evidence of a quite significant cross-sectional heteroskedasticity among bonds in the sample. The sample standard

Table 1
Number special bonds meeting selection criteria
In this table we show the number of bonds that have traded special for a number of days larger than [50, 100, 150, 200] and with a special repo rate larger than [50, 100, 150, 200] basis points in at least one trading day.

	Max value of 1 week repo spread			
Days on special	$50bp$	$100bp$	$150bp$	$200bp$
50	22	14	8	5
100	19	12	6	4
150	15	10	5	4
200	8	6	4	3

Table 2

Descriptive statistics of special bonds

The table reports descriptive statistics of repo spreads for the special bonds used in the empirical analysis. Values have been rounded to integers and expressed in basis points. Standard deviations are expressed in basis points at daily frequency.

Bond name	1*w*	1*m*	2*m*	3*m*	Bond name	1*w*	1*m*	2*m*	3*m*
OBL 6.75% 15 Sep 99	No. of obs: 116				OBL 5.125% 21 Nov 00	No. of obs: 103			
Average	26	25	25	23	Average	28	25	24	22
Standard deviation	20.79	8.91	6.71	5.85	Standard deviation	19.60	5.53	4.65	5.76
Min	3	11	12	12	Min	13	16	15	13
Max	113	62	53	50	Max	156	49	32	34
Autocorrel	0.886	0.776	0.692	0.676	Autocorrel	0.624	0.740	0.908	0.966
OBL 5.25% 21 Feb 01	No. of obs: 108				OBL 5.00% 21 May 01	No. of obs: 172			
Average	25	23	23	22	Average	54	48	46	44
Standard deviation	13.40	6.02	6.19	5.86	Standard deviation	48.12	32.47	28.14	26.91
Min	11	7	9	10	Min	14	12	13	14
Max	106	41	36	39	Max	220	141	120	116
Autocorrel	0.613	0.804	0.931	0.935	Autocorrel	0.954	0.977	0.975	0.983
OBL 5.00% 20 Aug 01	No. of obs: 224				OBL 8.75% 20 Aug 01	No. of obs: 224			
Average	21	24	28	30	Average	10	9	8	8
Standard deviation	32.37	26.50	28.70	33.33	Standard deviation	20.37	5.75	4.96	4.39
Min	3	4	2	5	Min	2	3	2	1
Max	182	135	146	164	Max	235	46	27	27
Autocorrel	0.950	0.937	0.977	0.988	Autocorrel	0.681	0.881	0.949	0.948
OBL 8.25% 20 Sep 01	No. of obs: 249				OBL 6.25% 26 Apr 06	No. of obs: 172			
Average	12	11	11	11	Average	21	23	24	24
Standard deviation	16.46	10.37	7.31	6.64	Standard deviation	21.31	18.36	16.14	15.46
Min	2	2	2	1	Min	4	5	6	6
Max	215	95	37	37	Max	136	90	62	57
Autocorrel	0.848	0.805	0.970	0.959	Autocorrel	0.823	0.921	0.952	0.957
OBL 6.00% 4 Jan 07	No. of obs: 314				OBL 6.00% 4 Jul 07	No. of obs: 266			
Average	13	15	17	16	Average	41	36	37	35
Standard deviation	16.37	10.42	10.47	10.32	Standard deviation	43.95	28.82	26.47	19.71
Min	0.3	3.5	0.7	0.2	Min	2	6	7	5
Max	136	60	62	58	Max	248	153	139	102
Autocorrel	0.797	0.867	0.889	0.913	Autocorrel	0.565	0.890	0.955	0.958
OBL 5.25% 4 Jan 08	No. of obs: 100				OBL 6.00% 20 Jun 16	No. of obs: 333			
Average	56	51	53	53	Average	23	21	20	19
Standard deviation	36.98	25.05	21.82	18.60	Standard deviation	15.56	11.13	8.90	7.79
Min	8	8	10	11	Min	5	4	4	4
Max	197	123	95	89	Max	108	80	62	57
Autocorrel	0.730	0.887	0.925	0.913	Autocorrel	0.593	0.809	0.903	0.926

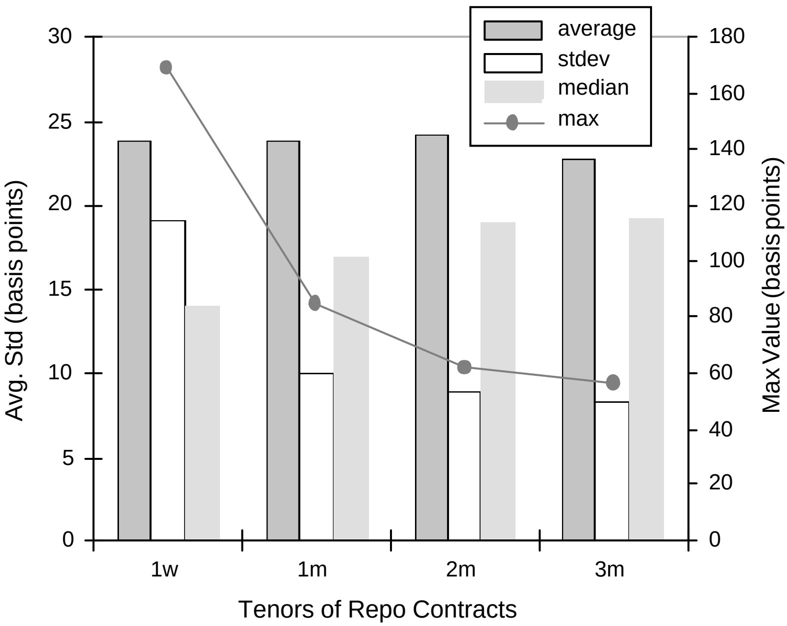

Fig. 1. Summary statistics of the special repo spread. The first three columns are the average, standard deviation and median of the special repo spread respectively. The unit of measure are in basis points on the left axis. The dotted line is the maximum value of the repo spread. The unit of measure are in basis points on the right axis.

deviation of the one week repo spread ranges from 13.40*bp* for OBL[9] 5.25% 21 February 2001, to 48.12*bp* for OBL 5% May 2001.

Third, on average the volatility curve of repo spread tends to be downward sloping, with the one week repo spread exhibiting the highest volatility. This phenomenon is documented in Fig. 1 that shows the median and the average (across different issues) of the repo spreads, the median volatility curve, and the median maximum value for repo spreads of different tenors (right scale). The shape of the volatility curve confirms that cases of exceptional specialness are indeed expected to be temporary.

Fourth, the term structure of median repo spread is upward sloping. The median two months repo spread is 19*bp*, about 30% higher than the median of the one week repo spread, which is equal to 14*bp* (see Fig. 1). Moreover, the cross-sectional distribution of repo spreads is very skewed: the median two months repo spread is about 20% smaller than the mean.

[9] German government bonds include "Deutsche Bundesrepublik" (ten, 15 and 30 year securities, commonly known as BUNDS), "Treuhandstalt" (ten year securities), "German Unity Fund" (ten year securities), "Bundesobligationen" (five year securities, known as OBL), "Treuhand-Obligationen" (five year securities, known as TOBL), "Bundesschatzanweisungen" (two year securities, commonly known as SCHATZ).

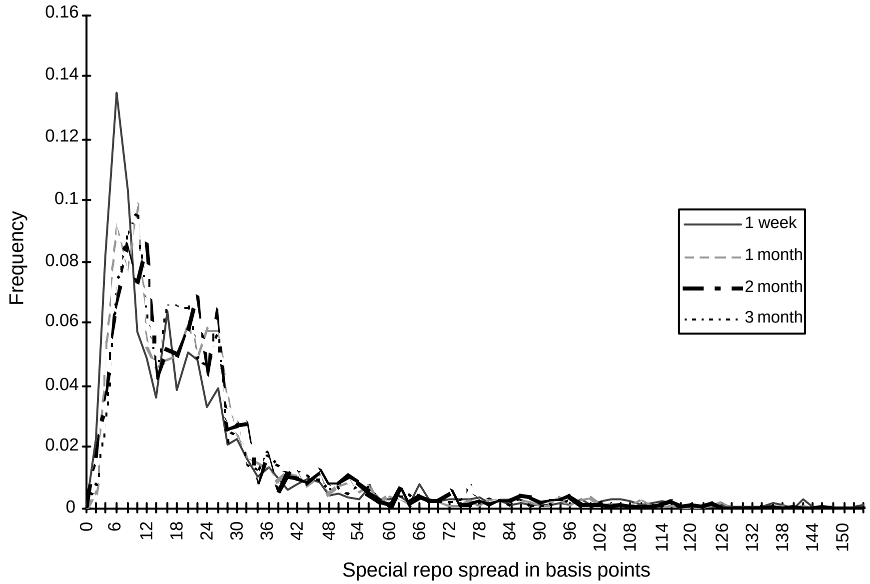

Fig. 2. Distribution of the special repo spread. Each line refers to the average repo spread for different tenors over time. The tenors are one week, one month, two months and three months.

Fig. 2 shows an estimate of the unconditional density function of repo spreads of contracts with different tenors and it indicates the large skewness in the distribution of repo spreads.

The conditional behavior of the slope is very rich. The slope can become as high as 100*bp* and the volatility of the slope is equal to 19.26*bp*, 17.94*bp* and 14.82*bp* for the three months minus one week, two months minus one week and one month minus one week, respectively. This volatility is quite important if one compares its magnitude to the level of the repo spread that on average, across all bonds on special, is about 23*bp* and it motivates the conditional empirical exercise.

Fifth, we explore whether those bonds that remained special for a small period of time have different characteristics than those bonds that remained special for a longer period of time. We split the sample in two parts, based on the length of time they remained special, and analyze whether any of the bond characteristics are found more frequently in any of the two subsets. The characteristics that we consider are: maturity, coupon rate, age, deliverability, whether the bond has been auctioned during the sample period, and whether the bond has been deliverable against the future. In terms of the first four characteristics, the bonds that remained special for less than 100 days are no different than the ones that remained special for more than 100 days: the distribution of these four characteristics is essentially the same in the two subsamples. However, there is a mild pattern that links the number of days that a bond remains special to the fact that the bond has been deliverable during the sample period. The bonds that have traded special less than 100 days are 1.7% more likely to be deliverable. The result is interesting since it is rare for a bond that is not deliverable to become special at all. The results suggest that if a bond that is nondeliverable becomes special, then it remains special for a longer period of time. However, due to the small number of nondeliverable bonds in the sample, the results are not statistically significant. Moreover, we learn that the auction cycle, while being an important reason for a bond to become special, is not exclusively associated with a short-term phenomenon of specialness: 25% of the bonds being auctioned during the sample continue to remain special well after 100 days from the auction. See Table 3.

Finally, the first order autocorrelation of repo spreads is substantially lower than for interest rates. Moreover, the autocorrelation is smaller for short tenor contracts (the median value for the one week repo spread is 79.67%). The small persistence in repo spreads, as opposed to interest rate levels, is clearly a desirable feature for our regression analysis.

We check whether there is any form of staleness between different segments of the curve. We find that in only 0.72% of the sample the one week special repo rate changes without a contemporaneous change in the long-term rate. This occurs for small movements of the one week repo spread, namely eight basis points on average. We can safely conclude that the data does not suffer from any serious form of staleness.

3.1. Jumps or a diffusion?

Since very little is known of the empirical properties of the special repo market, before studying the existence of a time varying liquidity risk premium, we analyze in

Table 3
Analysis of the length a bond remains special
This table presents summary statistics and regression analysis of the length of time a bond has been special as a function of its characteristics: coupon, age, maturity, whether the bond was deliverability against the future, and whether the bond was auctioned during the sample. In the top panel we present the means of the characteristics of two subsamples: the bonds that remained special more than 100 days and all the other bonds that remained special for a period of time less than 100 days. In the bottom panel we present the result of a regression of the length of time a bond has remained special onto the bond characteristics.

Sample statistics	No. of bonds	Coupon	Age	Maturity	Non-deliv.	Auctioned
> 100 days	43	6.14%	6.45	4.9	8.3%	25%
Others	23	5.80%	6.80	5.2	6.7%	23.3%
Regression analysis	Intercept	Coupon	Age	Maturity	Non-deliv.	Auctioned
Length time on special	120 (0.07)	8.62 (0.48)	− 2.01 (0.76)	− 0.004 (0.78)	158.39 (0.22)	27.94 (0.42)
R^2	16%					
F p-value	0.04					

more detail the stochastic characteristics of the repo spread. In this section, we ask the following questions: Is the special repo rate a diffusive stochastic process, or is it better described by a jump process? What is the extent to which specialness is driven by discontinuous jumps in the repo rate? In order to address this issue, we use a nonparametric technique to estimate the following generalization of the standard diffusion process for the repo spread:

$$\mathrm{d}L_t^i = \mu(L_t^i)\,\mathrm{d}t + \sigma(L_t^i)\,\mathrm{d}W_t + \mathrm{d}J_t, \tag{11}$$

where $\mathrm{d}J_t$ is a Poisson jump process with arrival intensity $\lambda(L_t^i)$ and jump size $y \sim \mathrm{N}(0, \sigma_y^2)$. In general, both the original intensity and the jump size may depend on the level of the repo spread. The moments of the infinitesimal changes of the process are defined as

$$M^p(l) = \lim_{\Delta \to 0} \frac{1}{\Delta} \mathrm{E}[(L_{t+\Delta}^i - L_t^i)^p | L_t^i = l], \tag{12}$$

$$M^1(l) = \mu(l),$$

$$M^2(l) = \sigma^2(l) + \lambda(l)\mathrm{E}_y(y^2),$$

$$M^3(l) = 0,$$

$$M^p(l) = \begin{cases} \sigma_y^2[(p-1)(p-3)\cdots 3 \times 1]\lambda(l) & \forall p > 3 \text{ even}, \\ 0 & \forall p > 3 \text{ odd}. \end{cases} \tag{13}$$

From the moment restrictions, it is clear that the first two moments cannot identify the presence of jumps. If one were to focus solely on the first two moments, a jump-diffusion process would be observationally equivalent to a pure diffusion process with local volatility equal to $\sigma^2(l) + \lambda(l)\mathrm{E}_y(y^2)$, as opposed to just $\sigma^2(l)$. However, if one considers higher order moments it is possible to identify the potential presence of jump components. A pure diffusion process has all the moments of the infinitesimal changes with $p \geqslant 3$ equal to zero. If jumps do affect the process, then they are responsible for nonzero third and fourth moments. Thus, one can use these higher order moments to estimate the extent to which jumps characterize the stochastic process of the repo spread. Let us consider a standard nonparametric estimator of the pth moment:

$$\hat{M}^p(l) = \frac{\sum_i K\left(\frac{L_{(i+1)\delta} - l}{h}\right)\left[\frac{L_{(i+1)\delta} - L_{i\delta}}{\Delta}\right]^p}{\sum_i K\left(\frac{L_{(i+1)\delta} - l}{h}\right)} \tag{14}$$

with Δ being the time interval between two consecutive observations of $\{L_{i\Delta}\}_{i=1}^N$ and $\Delta = T/N$ and h being the bandwidth of the kernel function $K(l)$. Then, an estimator of the jump components can be obtained sequentially as follows:

$$\hat{\sigma}_y^2 = \frac{\hat{M}^6(l)}{5\hat{M}^4(l)}, \tag{15}$$

$$\hat{\lambda}(l) = \frac{\hat{M}^4(l)}{3\hat{\sigma}_y^2}, \tag{16}$$

$$\hat{\sigma}(l) = \hat{M}^2(l) - \hat{\lambda}(l)\hat{\sigma}_y^2, \tag{17}$$

$$\hat{\mu}(l) = \hat{M}^1(l). \tag{18}$$

Johannes (2000) and Bandi and Nguyen (2000) provide a discussion of this approach and Bandi and Nguyen (2000) provide a derivation of the asymptotic distribution of the estimators. In Fig. 3 we compare the coefficients of the jump components for the level of the interest rate, i.e., the general collateral rate, and of the special repo spread. The estimation is performed using the entire panel data of the bonds.

The results of the estimation show that the stochastic process of the repo spread is mean reverting. However, from the plots in Fig. 3, the mean reversion is higher for highly negative values of the repo spread. When bonds are trading at a general collateral level, the local drift is negative but we cannot reject the null hypothesis that the local drift is zero. For large (negative) values of the specialness, the drift assumes the highest value. When we run a test of whether the drift is constant, we can reject the null hypothesis that the drift is independent of the level of the repo spread.

From an unconditional perspective, if we define $\sqrt{\sigma^2 + \lambda\sigma_y^2}$ the total volatility of the process, we find that the diffusive component of the one week repo spread, namely σ^2, is responsible for 83% of the total volatility. The jump component becomes less important for contracts with longer tenors. For a three months tenor contract the jump component constitutes about 9% of the total volatility, with respect to 17% for the one week repo spread. The results are not very dissimilar to the ones that can be obtained for the level of the general collateral repo rate. However, the main difference is that in the process of the level of the general collateral rate, jumps occur less frequently but when they occur they are larger in size. The value of σ_y is 35*bp* for the level of the general collateral, with respect to 20*bp* for the one week repo spread.

We find that the contribution of the jump process to the total volatility of the repo spread depends on the level of the repo spread. From the conditional nonparametric estimates, we learn that when the bond trades at a spread of about 50*bp* the total volatility is 20*bp*. The contribution due to the diffusion term is 15*bp*. Thus, for this level of the specialness, the jump component is about a third of the diffusion component and it counts for about 25% of the total volatility. The volatility of the jump component is nonlinear: it has the highest value for zero levels of the repo spread, it declines to its minimum level for a repo specialness of 25*bp* and then it increases for higher levels of the specialness. The volatility of the jump component is 19*bp* conditional on the repo spread being 100*bp*. These results confirm what can be

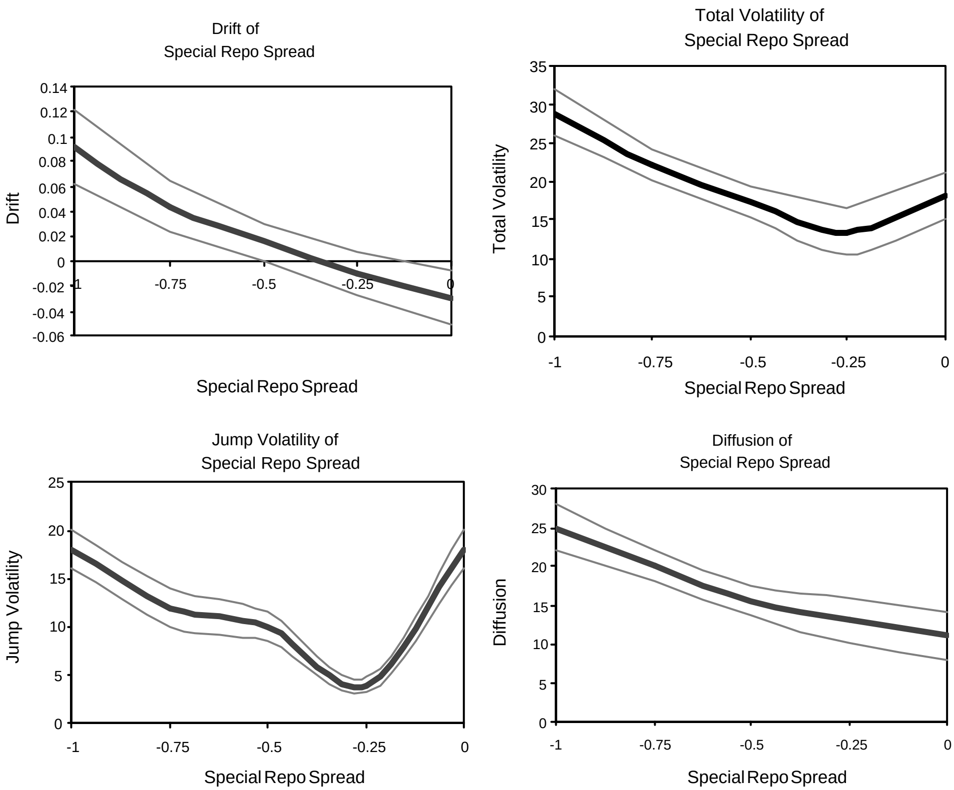

Fig. 3. Nonparametric estimates of the following mixed jump-diffusion process for the special repo spread $\mathrm{d}l_t = \mu(l_t)\,\mathrm{d}t + \sigma(l_t)\,\mathrm{d}W_t + \lambda\,\mathrm{d}J_t$. The second moment of the special repo spread is given by $\sigma^2(l_t) + \lambda^2\sigma_J^2(l_t)$. The first panel on the left displays $\mu(l_t)$. The right-top panel displays the total volatility defined as $\sqrt{\sigma^2(l_t) + \lambda^2\sigma_J^2(l_t)}$. In the two bottom panels the total volatility is decomposed in the volatility $\sigma_J(l_t)$ generated by the jump component (left hand side) and in the volatility $\sigma(l_t)$ due to the continuous-path diffusion process. The two thin bounds give the [10%, 90%] small sample confidence bounds. All volatilities are measured in basis points.

obtained by isolating the behavior of the specialness at the boundaries of the process. On the first day a bond goes on special, the repo spread jumps on average from zero to 23*bp*. On the last day in which a bond trade special, the average size of the jump is 18*bp*. Although the jump volatility is about half of the diffusive volatility, the frequency of these events, captured by the Poisson parameter λ, is small so that the process behaves approximately as a diffusion for interior levels of the special repo rate.

4. Empirical evidence

As a preliminary study, we first run a set of OLS regressions on each of the 43 individual bonds that have been on special at least 100 days. Then, we pool the data set in a single panel and we explore the spread expectation hypothesis using a simultaneous regression approach. The advantage of this approach is to reduce the small sample bias that would affect any individual regression and to increase the power of the tests.

4.1. Individual regressions

In order to investigate the ability of the slope of the term structure of repo spread, specific to bond i, to price changes in *long-term* (type A) and *short-term* (type B) repo spread, we estimate Eqs. (9) and (10) using ordinary least squares based on different pairs of short and long term-repo spreads sampled at daily frequency. Table 4 reports the results for repo spreads of different tenors, ranging from one week to three months. The point estimates are corrected for small sample bias and the standard errors are adjusted for overlapping observations. Both of these issues will be discussed in Section 5.

Based on the results reported in Table 4, the null hypothesis of the expectations hypothesis is strongly rejected, although it is rejected more mildly in the case of short-term regressions. Interestingly, this evidence is similar to the case of standard tests of the term structure of interest rates.

Betas are generally more than two standard errors away from one. Contrary to what is predicted by the expectations hypothesis, in all long-term tests we observe negative average beta coefficients (type A regressions). In regressions of type B, apart from three exceptions, the beta coefficients are always positive. In the case of regressions of type A, we reject the null hypothesis that β is equal to one with an average frequency of 88%, and we reject the null hypothesis that $\alpha = 0$ with a frequency of 10%.

With regards to regressions of type B, 84.5% of the times we reject the null hypothesis that the slope of term repo spreads price correctly future changes in short term repo spreads according to the expectations hypothesis, i.e., $\beta = 1$. For tests of type B, the frequency of rejection of the null hypothesis that $H_0{:}\alpha = 0$ is 18%.

Based on the results reported in Table 4, in the great majority of cases we find that the slope of the term structure of repo spreads greatly overestimates future changes in long term special repo spreads. High current term repo spreads versus current short-term repo spreads are not followed by a large increase in future short-term repo spreads.

Although the expectations hypothesis remains rejected in 84.5% of the cases, the extent of overestimation is lower in the case of pricing future short term tenor repo spreads (type B regressions). The average β is positive and in only in few occasions we find negative values. An analysis of the R^2 of tests A and B is another confirmation that the future specialness of repo contracts with long-term tenor is poorly anticipated by the current term structure. The slope of the repo spread curve tends to have a much higher explanatory power on changes in repo (spreads) contracts with short term tenor (average $R^2 = 30\%$), than on changes in repo (spreads) with long term tenor (average $R^2 = 16\%$).

Table 4
Individual regressions
This table reports the average results of individual regressions on all bonds in the sample. The average is computed across the 43 bonds. A1–B4 are the eight specifications of the expectation hypothesis, depending on the tenors of the contracts. The values of t-stat $H_0{:}\alpha = 0$ are the t statistics for a test of the nulll hypohesis $\alpha = 0$, and similarly for $\beta = 0$ and 1. The standard deviations are corrected for heteroskedasticity and autocorrelation using Newey-West (1987).
Type A regression:

$$L_i(t+\Delta t, T) - L_i(t, T) = \alpha + \beta\left[\frac{\Delta t}{T-(t+\Delta t)}\right][L_i(t, T) - L_i(t, t+\Delta t)] + \varepsilon_i.$$

Type B regression:

$$\sum_{s=1}^{n-1}\left(1-\frac{s}{n}\right)\Delta^s L_t = \alpha + \beta[L_t(t, T) - L_t(t, t+\Delta t)] + \varepsilon_t.$$

Term repo spreads combinations

Regression	*Short-term tenor*	*Long-term tenor*
A1 and B1	$\Delta t = 1$ week	$T = 1$ month
A2 and B2	$\Delta t = 1$ week	$T = 3$ month
A3 and B3	$\Delta t = 1$ month	$T = 2$ month
A4 and B4	$\Delta t = 1$ month	$T = 3$ month

	Avg α	Avg β	Avg R^2	Rej. rate $\alpha = 0$	Rej. rate $\beta = 0$	Rej. rate $\beta = 1$
A1	0.01	− 0.61	0.16	0.00	0.68	0.89
A2	− 0.17	− 1.79	0.11	0.07	0.64	0.86
A3	− 0.99	− 0.55	0.18	0.18	0.68	0.95
A4	− 0.46	− 0.78	0.18	0.14	0.59	0.82
B1	0.02	0.55	0.33	0.09	0.61	0.77
B2	− 0.63	0.57	0.46	0.41	0.70	0.75
B3	− 0.49	0.22	0.15	0.18	0.32	0.95
B4	− 0.55	0.33	0.25	0.36	0.48	0.91
Average	− 0.41	− 0.26	0.23	0.18	0.59	0.86

The results are surprising since they suggest that current forward repo spreads poorly reflect the future convenience yield generated by bonds that are on special. A positive (negative) slope of the term structure of repo spreads overestimate (underestimate) either the length of time a bond remains special, or the future special repo rate, or both.

We check whether there is a pattern the links the rejection to the length of time a bond remains on special. In particular, we run the same individual regressions for *all* 66 bonds, thus including the ones that have remained special for a period shorter than the maximum tenor of the repo contract. The point estimates are corrected for small sample bias and the standard errors are adjusted for overlapping observations. The results closely mirror the ones obtained for the data set with a longer time-series. The null of the expectations hypothesis is still strongly rejected and it is still the case

Table 5
Pattern analysis
This table reports the results of a regression of the rejection levels of the Spread Expectation Hypothesis onto a set of bond characteristics. The exogenous variables are: a constant, the bond maturity, the bond age, a dummy that indicates whether the bond was *not* deliverable against the futures, a dummy that indicates whether the bond was auctioned during the sample period, the maximum level of the spread, and the number of days on special. The endogenous variable is a dummy that takes values from 0 to 8 depending on the number of times $H_0{:}\beta = 1$ is rejected at the 5% confidence level. When the endogenous variable is equal to 0, the expectation hypothesis is never rejected in all eight tests presented in Table 3 (type A and B regressions each computed with respect to four different tenors).

Rejection intensity (0–8) of $H_0{:}\beta = 1$	Constant	Maturity	Age	Non-deliv.	Auction	Max spread	Days
Parameter	8.96	− 0.09	− 0.10	2.85	− 1.78	− 0.02	− 0.002
t-stat	7.66	− 0.78	− 0.86	1.06	− 2.23	− 1.87	− 0.51
R^2	29.78%						

that the rejection is milder in the case of short-term regressions. In the case of regressions of type A, we reject the null hypothesis that β is equal to one with an average frequency of 88%. With regards to regressions of type B, the rejection frequency of the null hypothesis $H_0{:}\beta = 1$ is 83.25%.

In order to take full advantage of the richness of the data set, we study the pattern of the rejection as a function of the characteristics of the bonds. We regress cross-sectionally the intensity[10] of the rejection onto six characteristics of the bonds: maturity, age, deliverability against the future, an indicator function that takes the value of one if the bond has been auctioned during the sample period, maximum spread, days on the special. The results are summarized in Table 5.

The results of the tests show that the deliverability against the futures is not significant in explaining the rejection of the spread EH. The same applies to the other characteristics with the exception of the auction cycle dummy. Auction cycles are related to the pattern of specialness, as documented by Fisher and Gilles (1996), Duffie (1996), and Jordan and Jordan (1997). The results of the test statistics indicate that for these bonds the expectation hypothesis of repo spreads are rejected less frequently. Given this result, we decide to study more closely the extent of this effect. We stratify the sample conditioning with respect to the fact that the bond has been auctioned during the sample period. Table 6 presents the results. For bonds that have been subject to auction during the sample period, the null hypothesis that $\beta = 1$ is rejected less frequently, but the average rejection frequency is still a sound 64% compared to 86% for the entire sample. Moreover, the frequency of rejection of $H_0{:}\alpha = 0$ goes from 11.63% to 1.79%. Both these two results may be an indication of a higher predictability of the specialness of bonds subject to auction cycle. However, the results of the test on $H_0{:}\beta = 0$ implies that the expectation hypothesis is still rejected even for bonds that are been auctioned during the sample period.

[10] We proxy the intensity of the rejection with a dummy that takes a value from zero to eight depending on the number of times $H_0{:}\beta = 1$ is rejected at the 5% confidence level.

Table 6
Auction cycle effect
This table reports the average rejection frequency, across the 8 tests A1–B4 of the expectation hypothesis conditional on the bond having been auctioned during the sample period.

	H_0:$\alpha = 0$	H_0:$\beta = 1$
Full sample	11.63%	86%
Auctioned bonds	1.79%	64%

The slope coefficient of the dummy variable that captures the nondeliverability of the bond against the future is negative. Thus, bonds that have the potential of becoming cheapest-to-deliver are also the ones for which the spread expectation hypothesis is rejected more strongly. However, the estimator is not statistically significant from zero.

4.2. From a time series to a simultaneous equations approach

Are the previous results due to a small sample issue? After all, each bond that occasionally trades at a special collateral rate represents an individual story, with all its market microstructure idiosyncrasies. Testing that the expectations hypothesis holds for each and every bond might be both too demanding and also costly in terms of power of the test statistics. In order to address this issue, we decide to pool the 12 bonds of the restricted data set in a single panel and test if the regression coefficients are equal across all regressions. By doing so, we ask if the current term structure of repo spreads anticipate future repo specialness, at least *on average over all bonds*.

This technique, that involves testing the spread expectations hypothesis on different securities of a given asset class at the same time, represents an interesting exercise in itself. It is made possible by the nature of our data set that provides all term repo spreads on all individual bonds at any point in time. The obvious advantage of this approach is to increase dramatically the number of independent observations with respect to the previous individual regression approach (on average from 160 to almost 2000). We run Monte Carlo simulations and find that the simultaneous regression approach is greatly beneficial in improving the small sample properties the individual regression approach. We refer to Section 5 for a detailed discussion.

We pool our cross-sectional and time-series data to form a panel data of bonds that have traded at a special collateral rate over our sample period. We use an estimation and testing approach similar to Zellner's seemingly unrelated regression (SUR) to study our testable restrictions. In complete analogy to the case considered above, we consider four type A and four type B dynamic restrictions implied by the expectations hypothesis. For each expectations hypothesis restriction (we omit the relevant index for notational convenience), we denote respectively by $\mathbf{y}_i$ and $\mathbf{X}_i$ the dependent variable vector and independent variable matrix for bond *i*. The dimension of $\mathbf{y}_i$ and $\mathbf{X}_i$ are respectively $(T \times 1)$ and $(T \times 2)$ where T denotes the number of trading days in our sample (348). Since each security traded at a special collateral rate only for a subset of the trading days in the sample, both $\mathbf{y}_i$ and $\mathbf{X}_i$ arrays are filled with zeros. We denote by n the number of securities in the sample (12).

The general formulation of the unrestricted model is

$$\underset{nT\times 1}{\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_n \end{bmatrix}} = \underset{nT\times 2n}{\begin{bmatrix} \mathbf{X}_1 & 0 & 0 & 0 \\ 0 & \mathbf{X}_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \mathbf{X}_n \end{bmatrix}} \underset{2n\times 1}{\begin{bmatrix} \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \\ \vdots \\ \boldsymbol{\beta}_n \end{bmatrix}} + \underset{nT\times 1n}{\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}}. \tag{19}$$

If we impose the constraint that both the intercept terms and slope coefficients are equal across different bonds, i.e., $\boldsymbol{\beta}_1 = \boldsymbol{\beta}_2 = \cdots = \boldsymbol{\beta}_n$, we can rewrite the system as

$$\underset{nT\times 1}{\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_n \end{bmatrix}} = \underset{nT\times 2}{\begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \vdots \\ \mathbf{X}_n \end{bmatrix}} \underset{2\times 1}{\boldsymbol{\beta}} + \underset{nT\times 1}{\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}} \tag{20}$$

or in more compact form as

$$\mathrm{Y} = \mathrm{X}\boldsymbol{\beta} + \varepsilon. \tag{21}$$

The restricted generalized least square estimator of $\boldsymbol{\beta}$ is given by

$$\hat{\boldsymbol{\beta}} = [\mathrm{X}'\boldsymbol{\Omega}^{-1}\mathrm{X}]^{-1}\mathrm{X}'\boldsymbol{\Omega}^{-1}\mathrm{Y}. \tag{22}$$

If the true error variance covariance matrix $\boldsymbol{\Sigma}$ were known, the weighting matrix $\boldsymbol{\Omega}^{-1}$ can be calculated as

$$\boldsymbol{\Omega}^{-1} = \boldsymbol{\Sigma}^{-1} \otimes I, \tag{23}$$

where I is a $(T \times T)$ identity matrix and the generic element σ_{ij} of $\boldsymbol{\Sigma}$ is given by $\mathrm{E}[\varepsilon_i\varepsilon_j]$.

A consistent estimate of the covariance matrix $\hat{\boldsymbol{\Sigma}}$ can be obtained from the estimated residuals of a first stage OLS on Eq. (21). This is equivalent to run OLS regressions bond by bond, as we did in Section 5. The reason is that no allowance is made to the contemporaneous covariance between errors and that the $\boldsymbol{\beta}_i$ coefficients are free to vary across bonds. Since the endogenous variables are overlapping, we use the Newey-West (1987) autocorrelation and heteroskedasticity consistent estimators of the covariance matrix.

The estimated residuals of different bonds $\hat{\varepsilon}_i$ and $\hat{\varepsilon}_j$ are typically only partially overlapping. In order to gain efficiency, instead of a standard sample covariance matrix, the estimator for $\hat{\sigma}_{ij}$ is obtained as $\hat{\rho}_{ij}\hat{\sigma}_i\hat{\sigma}_j$, where the correlation coefficient is based on the overlapping observations while $\hat{\sigma}_i$ and $\hat{\sigma}_j$ are based on all observations available on the individual repo spreads. Equivalently, the generic element of the lagged covariance matrix is $\hat{\Sigma}_\tau$ is constructed as $\hat{\rho}_{ij}(\tau)\hat{\sigma}_i\hat{\sigma}_j$.

An estimate of the variance covariance matrix is given by

$$\hat{\boldsymbol{\Sigma}} = \hat{\Sigma}_0 + \sum_{\tau=1}^{q}\left(1 - \frac{\tau}{1+q}\right)(\hat{\Sigma}_\tau + \hat{\Sigma}'_\tau). \tag{24}$$

The appropriate number of lags, q, is chosen so as to account for the spurious autocorrelation induced by overlapping endogenous variables. It is set to be equal to the average number of trading days between t and $t + \Delta t$ for regressions of type A and to the average number of trading days between $t + \Delta t$ and T for regressions of type B.

An aspect that makes the econometrics of this problem nonstandard is that our time series have different numbers of observations, so that the observations are only partially overlapping. Since the vectors of residuals $\hat{\varepsilon}_i$ have different numbers of observations, there is no guarantee that the Newey-West corrected sample estimator $\hat{\boldsymbol{\Sigma}}$ is positive semi-definite.[11] The reason is simple: each sample lagged covariance matrix $\hat{\Sigma}_\tau$ is not a quadratic form. To cope with this problem, we apply a simple and intuitively appealing Bayesian adjustment to our Newey and West (1987) sample covariance matrix. Let us define $\hat{\boldsymbol{\Sigma}}_\mathrm{d} = \hat{\boldsymbol{\Sigma}} \otimes I$ to be the positive definite diagonal matrix of sample variances. This would be a consistent estimator if repo spreads were purely idiosyncratic and uncorrelated across different bonds. We then consider a shrinkage estimator obtained as a convex combination[12] of $\hat{\boldsymbol{\Sigma}}$ and $\hat{\boldsymbol{\Sigma}}_\mathrm{d}$ as

$$\hat{\boldsymbol{\Sigma}}_\mathrm{B} = w\hat{\boldsymbol{\Sigma}} + (1 - w)\hat{\boldsymbol{\Sigma}}_\mathrm{d}. \tag{25}$$

The weight w is chosen as the minimum value that gives a strictly positive-definite estimator $\hat{\boldsymbol{\Sigma}}_\mathrm{B}$ of the covariance matrix. This procedure maximizes the weight attributed to the sample covariance matrix while imposing the resulting estimate to meet the minimal requirement for the estimator to be a covariance matrix, i.e., to be positive semi-definite. Note that asymptotically (i.e., as the number of overlapping observation for any bonds i and j converge to the total number of observations) this procedure produces an estimate of $\boldsymbol{\Sigma}$ that puts no weights to the prior and fully exploits the information in the data set.

Table 7 reports the results of the simultaneous equation approach. The results confirm the impression obtained in the previous section. In both regressions of type

[11] An extreme solution would be to restrict the estimation of $\hat{\boldsymbol{\Sigma}}$ only on the trading days in which all bonds in the sample were trading special, thus solving the problem associated with different number of observations. However, this is unfeasible since the intersection of the observation period for all bonds is the null set, as there is no time period in which *all* bonds were active on the special repo market. Another technique has been recently suggested by Bansal and Dahlquist (1999) in the contest of data sets with missing data. They propose to treat missing observation as observations generating deviations from the testable restrictions equal to zero. In our framework, this is equivalent to assume that all bonds have been auctioned before the beginning of the sample period and that they were exactly consistent with the expectation hypothesis.

[12] This approach is similar to the Bayesian technique suggested by Ledoit (1996) for ill-conditioned semi-positive definite covariance matrix. Ledoit (1996) studies the asymptotic properties of a shrinkage estimator obtained as a linear combination of the sample covariance matrix and the identity matrix. Unfortunately, if the sample covariance matrix is not semi-positive definite in the first place, as in our case, Ledoit's shrinkage estimator is not guaranteed to be semi-positive definite.

Table 7
Simultaneous equations regressions
This table reports regression results based on a simultaneous equation approach. Let $L^i(t,T)$ be the spread between general collateral and special repo rates, $L^i(t,T) \equiv R(t,T) - S^i(t,T)$. Based on the spread expectation hypothesis, the relation between the current shape of the term structure of special repo spreads, specific to bond i, and future changes in the special repo rates of bond i is given by
Type A regression:

$$L_i(t+\Delta t, T) - L_i(t,T) = \alpha + \beta\left[\frac{\Delta t}{T-(t+\Delta t)}\right][L_i(t,T) - L_i(t,t+\Delta t)] + \varepsilon_i,$$

Type B regression:

$$\sum_{s=1}^{n-1}\left(1-\frac{s}{n}\right)\Delta^s L_i = \alpha + \beta[L_i(t,T) - L_i(t,t+\Delta t)] + \varepsilon_i,$$

where the index i ($i = 1, \ldots, 12$) denotes different special bonds and Δt denotes different time steps. Type A regression refers to the ability of the current term structure of repo spreads to anticipate future long-term tenor special repo rates. Type B regression refers to short-term tenor contracts.

Regression type	a (std error)	β (std error)	R^2	$H_0{:}\alpha = 0$ p-value	$H_0{:}\beta = 0$ p-value	$H_0{:}\beta = 1$ p-value
Type A regression						
$\Delta t = 1$ week	0.79	− 0.11	0.00	0.11	0.27	0.00
$T = 1$ month	(0.50)	(0.10)				
$\Delta t = 1$ week	− 0.05	0.28	0.00	0.80	0.06	0.00
$T = 3$ months	(0.20)	(0.15)				
$\Delta t = 1$ month	− 0.87	− 0.08	0.00	0.41	0.53	0.00
$T = 2$ months	(1.06)	(0.13)				
$\Delta t = 1$ month	1.46	− 0.59	0.00	0.41	0.08	0.00
$T = 3$ months	(1.76)	(0.34)				
Type B regression						
$\Delta t = 1$ week	0.42	0.83	0.30	0.57	0.00	0.00
$T = 1$ month	(0.74)	(0.04)				
$\Delta t = 1$ week	0.36	0.94	0.46	0.83	0.00	0.13
$T = 3$ months	(1.64)	(0.04)				
$\Delta t = 1$ month	− 0.43	0.46	0.07	0.41	0.00	0.00
$T = 2$ months	(0.53)	(0.06)				
$\Delta t = 1$ month	− 0.64	0.55	0.16	0.42	0.00	0.00
$T = 3$ months	(0.79)	(0.05)				

A and B, the point estimate of the intercept is never significantly different from zero. However, the slope coefficient is significantly different from one, in contrast with what is predicted by the EH. We reject the null hypothesis of a unit slope coefficient in all type A regressions, at any confidence level. In regressions of type B, we reject the null hypothesis of a unit slope coefficient in three out of four cases. For long tenor repo spreads (regression of type A), forward repo spreads fail to anticipate future special repo rates to the extent that in three cases out of four we even find negative slope coefficients. For short term tenor repo spreads, the point estimate of the slope coefficient is always positive (regression of type B). When we consider long term tenor repo spreads, in two cases out of four we cannot reject the null that forward repo spreads do not anticipate at all, i.e., $\beta = 0$, future special repo spreads.

The lack of ability of current forward repo spreads to anticipate future special repo spreads for long term tenor contracts is confirmed by the R^2 of regressions of type A. The explanatory power of the slope of the repo spread curve on subsequent repo spread movements is virtually zero for the long-term spreads equation (the average R^2 is close to zero). This contrasts with an R^2 that can be as high as 46% (on average 25%) for the short term tenor repo spreads equation (regression of type B). We check the robustness of the results by running a panel data regression using the SUR methodology also on an augmented dataset that also includes bonds that have remained special for a period shorter that three months, i.e., the maximum tenor of the repo contract. The results are overall similar to the results based on the original time series based on a longer time series. If anything, the results based on the extended data set suggest a slightly stronger rejection of the EH.

5. Monte Carlo analysis

5.1. Small sample bias

It is well known that any regression analysis based on persistent explanatory variables can suffer from small sample bias. Suppose that the value of the dependent variable is positively related to future realizations of the short-term spread. If the independent variable (the slope of the term structure of repo spreads) is negatively correlated with the short-term spread, then persistence in short-term repo spread may cause an upward bias in the estimated value of the slope coefficients. This issue has been studied by Stambaugh (1986), Mankiw and Shapiro (1986), and Elliott and Stock (1994) and it has been analyzed in the context of the expectations hypothesis of interest rate by Bekaert et al. (1997). In order to interpret the results of our exercise on repo spreads, it is important to understand the extent to which we can rely on asymptotic test statistics. What is the size of the small sample bias that is present in our results? And also, what is the extent of bias in tests of type A versus tests of type B for repo spreads?

Second, the structure of our dependent variable is responsible for overlapping residuals. The number of completely independent measures of the forecasting ability of the slope of the term structure of repo spreads are inversely proportional to the long-term tenor of the repo contract used to run the regressions. This can affect the power of our tests.[13] Moreover, the Newey and West (1987) adjustment for autocorrelated and heteroskedastic residuals relies on asymptotic arguments and its effectiveness in a finite sample is to be explored, especially when the degree of overlap is not trivial with respect to the sample size. What is the power of tests based on the simultaneous equation approach, given the structure of our data set?

[13] This issue is not specific just to our study. The same problem is common in any other test of the expectations hypothesis and also in tests of long-term predictability.

Third, aside from the issue of the power of the test, is our simultaneous regression approach able to significantly reduce the small sample bias, or is the SUR bias close the average bias of the individual regressions?

We address these three questions by deriving the small sample distribution of our estimators and test statistics using well designed Monte Carlo simulations.[14] For each regression type, we implement four sets of Monte Carlo experiments, depending on the sampling frequency and the tenor of the repo contract that we simulate. Each experiment consists of four steps: (1) estimation of structural parameters based on an auxiliary model, (2) simulation of a panel data of state variables, i.e., one week and one month repo spreads, given the structural parameters estimated in step 1, (3) simulation of the endogenous variables, i.e., term-repo spreads, conditional on the expectations hypothesis being satisfied, (4) estimation of the pricing restrictions for each simulated panel.

Step 1: We estimate the one week and one month repo spread process. In our regression analysis the short-term repo spread is the one week (regressions A1, A2, B1, B2) and one month (regressions A3, A4, B3, B4). We postulate a simple AR(1) specification for the repo spread process and estimate the parameters of the model using OLS on an individual bonds basis. We do not consider specifications with time-varying volatility since these usually give rise to time varying risk premia, which are inconsistent with our null hypothesis. Our auxiliary model is

$$L^{ij}(t, t+\Delta t) = \mu^{ij} + \rho^{ij} L^{i}(t - 1d, t+\Delta t) + \sigma^{ij}\varepsilon_{L}^{ij}(t), \tag{26}$$

where i varies among bonds, j denotes either one week or one month and d is one trading day. The residuals are assumed to be independently and identically distributed.

Note that richer specifications of the repo spread process could have been proposed, such as one that includes conditional heteroskedastic terms. However, any model with a time-varying volatility of the repo spread would, by construction, contradict the EH, since it necessarily induces a time-varying risk premium and time-varying Jensen's adjustments. We follow Bekaert et al. (1997) and adjust the estimated coefficients for small sample bias.[15] We obtain a consistent estimate of the variance covariance matrix of residuals, Σ^{j}_{MC}, computed using the technique described in Section 6.1.[16] Table 8 portrays the results of this analysis. In the interest of brevity the two variance–covariance matrices are not reported here.

Step 2: Given the estimated parameters, we then generate sample paths for the one week and one month repo spread process using a traditional Monte Carlo technique.

[14] Bekaert et al. (1997) discuss the issue of the small sample bias in the contest of the EH of the term structure of interest rates. Using a Monte Carlo approach, they show that the size of bias can be very large. Campbell and Shiller (1991) use Monte Carlo experiments to document the small sample sensitivity of estimates of the standard errors to overlapping residuals.

[15] Bekaert et al. (1997) show that, for a AR(1) process, the bias adjusted values of μ, ρ and σ are given by $\mu = \hat{\mu}(1-\rho)/(1-\hat{\rho})$; $\rho = (\hat{\rho} + 1/T)/(1 - 3/T)$; $\sigma = \hat{\sigma}[(1-\rho^2)/(1-\hat{\rho}^2)]^{1/2}$, where $\hat{\mu}$, $\hat{\rho}$ and $\hat{\sigma}$ are the sample estimates of the parameters and $\mathrm{E}(\hat{\rho}) - \rho = -[(1+3\rho)/T] + \mathrm{O}(1/T^2)$.

[16] As the sample variance covariance matrix turned out to be positive semi-definite no Bayesian Adjustment was required. We calculated Σ^{j}_{MC} at lag 0.

Table 8
AR(1) estimation results
This table reports results from the estimation of an AR(1) process for the repo spreads associated to the twelve bonds in our data set:

$$L^i(t+\Delta t, T+\Delta t) = \mu + \rho L^i(t,T) + \varepsilon_{t+\Delta t}.$$

Repo spread tenor	$L(t,t+1w)$				$L(t,t+1m)$			
	μ	ρ	σ	R^2	μ	ρ	σ	R^2
OBL 6.75 15 Sep 99	2.01	0.92	8.56	0.76	4.68	0.81	5.46	0.54
OBL 5.25 21 Nov 00	9.50	0.66	14.80	0.39	5.35	0.78	3.49	0.55
OBL 5.25 21 Feb 01	8.80	0.64	10.27	0.38	2.98	0.86	9.32	0.65
OBL 5 21 May 01	1.57	0.97	11.25	0.91	0.20	0.99	2.88	0.96
OBL 5 20 Aug 01	1.33	0.94	11.51	0.85	0.66	0.97	4.53	0.91
DBRUF 8.75 21 Aug 01	2.31	0.76	13.16	0.56	1.39	0.83	3.00	0.69
DBR 8.25 20 Sep 01	4.94	0.52	5.81	0.67	1.85	0.80	3.41	0.85
DBR 6.00 26 Apr 06	2.54	0.87	10.08	0.74	0.65	0.97	4.53	0.90
DBR 6.00 4 Jan 07	2.78	0.78	10.34	0.58	1.40	0.90	4.90	0.75
DBR 6.00 4 Jul 07	3.39	0.92	17.36	0.82	1.51	0.96	8.21	0.89
DBR 5.25 4 Jan 08	13.20	0.77	24.71	0.48	3.85	0.92	11.00	0.70
DBR 6.00 20 Jun 16	1.96	0.91	6.38	0.81	1.33	0.94	9.93	0.85

At each trading day, we simulate a vector of 12 independent normally distributed random numbers N, and turn it into a correlated random number vector $N^* = CN$ where C is the Cholesky factorization matrix of the sample correlation matrix P such that $P = CC'$. We then generate values for the one week and one month repo spread based on Eq. (26). Notice that we generate values for any given time series of repo spread only if the bond was special at the corresponding trading days in our observed sample. By doing so, we create a setting with exactly the same unequal number of observations for any time series as in our sample. This enables us to investigate the properties of our econometric procedure in an environment that closely matches the feature of our original sample.

Step 3: Conditional on the expectations hypothesis being satisfied, we compute the implicit term structure of repo spread at different tenors by iterating Eq. (26), so that

$$\begin{aligned} \mathrm{E}_t[L(t+nd, t+nd+\Delta t)] &= \left(\sum_{i=1}^{n} \rho^{n-1}\right)\mu + \rho^n L(t, t+\Delta t) \\ &= \frac{\rho^{n-1}}{\rho - 1}\mu + \rho^n L(t, t+\Delta t). \end{aligned} \tag{27}$$

From Eq. (8), the date-T term-repo spread that is consistent with the expectations hypothesis is given by the arithmetic average of the current and expected repo spreads:

$$L^{\mathrm{eh}}(t,T) = \frac{L(t,t+\Delta t) + \sum_{i=1}^{n-1} \mathrm{E}_t[L(t+id, t+id+\Delta t)]}{n} \tag{28}$$

with $n = [T - (t + \Delta t)]/\Delta t$. For any Monte Carlo trial, we generate an artificial multi-dimensional data set that is, by construction, consistent with the EH.

Step 4: For every artificial data set we run the same kind of type A and B regression that we have presented in Section 6, adopting the same Bayesian shrinkage estimator of the covariance matrix. We run 500 trials for each type of regression. Having estimated regression parameters, we check for spurious rejection of the expectations hypothesis caused by biases in coefficient or standard error values following Campbell and Shiller (1991).

We calculate the ratio of Monte Carlo trials in which negative deviations form one of the estimated beta coefficient divided by its standard errors is higher than the corresponding empirical estimate. This indicator tells us how frequently we would have rejected the spread EH (when the data generating process obeys to it) more strongly than in our empirical exercise.

Table 9 reports results from the Monte Carlo experiment. The results show that the upward bias in the slope coefficient is quite modest (the average bias across all regression types is 0.02623) and it does not alter significantly the conclusion from the regression analysis. Moreover, the size of the bias is of comparable magnitude for both regressions of type A and B. This result is particularly important in the light of the different outcomes of the two types of regressions. The results cannot be simply related to distortions generated by small sample bias.

Table 9
Monte Carlo analysis
This table reports simultaneous regression results based on simulated repo spreads under the null hypothesis that the spread expectation hypothesis holds at every tenor. The column labeled frequency of over-rejection is the frequency of the simulated t-statistics that the slope coefficient is equal to one is higher than the t-statistics obtained on the actual sample. A value equal to 0.002 means that the t-value from the Monte Carlo paths, simulated under the expectation hypothesis, is higher than the t-value obtained from the actual sample in one out of five hundred simulations. A small number indicates that the likelihood of observing levels of t-statistics as observed in the empirical data is indeed small, if the null hypothesis that the spread expectation hypothesis is satisfied, even taking into account the small sample properties of the data and the procedure used to estimate the standard errors. q is the number of lags in the Newey-West correction for the covariance matrix.

Regression type	β^0	β^1	R^2	Frequency of over-rejection
A1 $_{q=5}$	0.00157	1.02447	0.13	0.000
A2 $_{q=5}$	− 0.00145	1.02413	0.11	0.000
A3 $_{q=20}$	0.03936	1.02446	0.11	0.000
A4 $_{q=20}$	0.00859	1.01724	0.13	0.002
B1 $_{q=15}$	0.06724	1.03845	0.47	0.000
B2 $_{q=55}$	− 0.23700	1.03115	0.70	Null accepted
B3 $_{q=20}$	0.04669	1.01145	0.34	0.000
B4 $_{q=40}$	0.13926	1.03891	0.48	0.000

Consistently with the results of Section 6.1, the explanatory power of type B regression is much higher than the one of type A. Remarkably, the Monte Carlo analysis shows that the frequency with which we would have rejected more strongly the null hypothesis with artificially generated data (by construction consistent with the spread EH) is virtually zero. This suggests that our Bayesian adjustment for the variance–covariance matrix has proven to be effective.

The absolute value of the bias is significantly lower than the value obtained by Bekaert et al. (1997) in their interest rate analysis. This is due to two main reasons. First, repo spreads are less autocorrelated than interest rates levels. Second, the simultaneous regression technique, that we use in this paper, effectively expand the sample size cross-sectionally. As the bonds in the sample are not perfectly correlated, this procedure has an analogous effect to a time-series increase in the number of actual observations that would reduce the small sample bias. Bekaert et al. (1997) use a sample of 524 observations and find a significantly upward bias in the slope coefficient. In our case, the average number of observations per bond is below 200. However, since we pooling all information in a single panel with a common slope coefficient, it is like working with a data set effectively larger by one order of magnitude.

One way to see this effect is by comparing the average size of the bias for the coefficient estimated on an individual bond basis with the bias based on the simultaneous equations approach. Table 10 reports the results. We denote respectively by $\hat{\beta}_{\text{sur}} - \beta_0$ and $\hat{\alpha}_{\text{sur}} - \alpha_0$ the bias in the slope and intercept coefficient estimated through our SUR approach and $E\hat{\beta}_{\text{ols}} - \beta_0$ and $E\hat{\alpha}_{\text{ols}} - \alpha_0$ the average biases resulting from the individual OLS regressions.

The average bias obtained with OLS is clearly much more serious than the one affecting the simultaneous regression approach. It is higher for type A regressions, and particularly significant for A3 and A4, that are based on the more persistent one month repo spread process. For these regressions, the average estimate of the slope coefficient is almost twice the correct one. For individual OLS regressions, the small sample bias is positive. Since the sample estimates of the slope is already significantly less than one, this would imply an even stronger rejection of the expectations hypothesis for the term structure of repo spreads.

The simultaneous regression approach is less sensitive to the small sample bias by more than an order of magnitude. In order to give a visual feeling of the extent to which the simultaneous equation approach solves the problem in our case, Fig. 4 plots the empirical distribution of the slope coefficient in the simultaneous equation approach and the average OLS β estimate in the individual regression approach (regression of type A4).

5.2. Measurement error

As pointed out by Mankiw and Shapiro (1986), Campbell and Shiller (1991), and Bekaert et al. (1997), among others, the estimation of the slope coefficient for Eqs. (9) and (10) can be affected by measurement error in the data.

Table 10

Bias comparison of individual and simultaneous regressions

In this table we report the difference in small sample bias of the estimated coefficients based on the individual regression approach and on the simultaneous approach. $E\hat{\beta}_{ols} - \beta_0$ is the difference between the average estimate of the slope coefficients from their true value, obtained from individual regressions based on repo spreads simulated under the spread expectation hypothesis. $\hat{\beta}_{sur} - \beta_0$ is the small sample bias in a simultaneous equation approach when the repo spreads are pooled in a panel. Similarly, we report results for the intercepts, here α.

	Individual regressions		Simultaneous equations	
Regression type	$E\hat{\beta}_{ols} - \beta_0$	$E\hat{\alpha}_{ols} - \alpha_0$	$\hat{\beta}_{sur} - \beta_0$	$\hat{\alpha}_{sur} - \alpha_0$
A1 $_{q=5}$	0.210	0.316	0.024	0.002
A2 $_{q=5}$	0.226	0.051	0.024	− 0.001
A3 $_{q=20}$	0.989	1.339	0.024	0.039
A4 $_{q=20}$	0.929	1.259	0.017	0.009
B1 $_{q=15}$	0.150	0.160	0.038	0.067
B1 $_{q=55}$	0.116	0.568	0.031	− 0.237
B1 $_{q=20}$	0.491	0.725	0.011	0.047
B1 $_{q=40}$	0.450	1.234	0.039	0.060

Whatever the reason, the estimated slope coefficients of the pricing restrictions from the EH will not be consistent when data on the long- and/or short-term rate contain noise. The intuition is simple. The time t measurement error features both at the right and left hand side of the pricing restriction, but with the opposite sign. This generates a downward asymptotic bias. It should be noticed, however, that the higher the autocorrelation in the noise, the smaller the extent of the downward bias.

Suppose that the observed values of the short and long tenor repo spread $\hat{L}^i$, specific to bond i, are given by their true counterpart plus a serially uncorrelated (and orthogonal to the true repo spread value) noise:

$$\hat{L}^i(t, t+\Delta t) = L^i(t, t+\Delta t) + \varepsilon^i_{t+\Delta t}, \tag{29}$$

$$\hat{L}^i(t, T) = L^i(t, T) + \varepsilon^i_T, \tag{30}$$

$$\begin{bmatrix} \varepsilon^i_{t+\Delta t} \\ \varepsilon^i_T \end{bmatrix} \sim \text{IID}\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^i_{t+\Delta t} & \sigma^i_{(T,t+\Delta t)} \\ \sigma^i_{(T,t+\Delta t)} & \sigma^i_T \end{bmatrix} \right\}. \tag{31}$$

In the *univariate case*, Bekaert et al. (1997) provide analytical formulas for the bias in the single regression coefficient for pricing restrictions of type A and B. If we let $\kappa = [T - (t+\Delta t)]/T$, the slope coefficient is biased away from one and it converges asymptotically to

$$\text{plim}(\beta^{\text{A}}) = 1 - b_\kappa, \quad \text{plim}(\beta^{\text{B}}) = 1 - \frac{b_\kappa}{\kappa}, \tag{32}$$

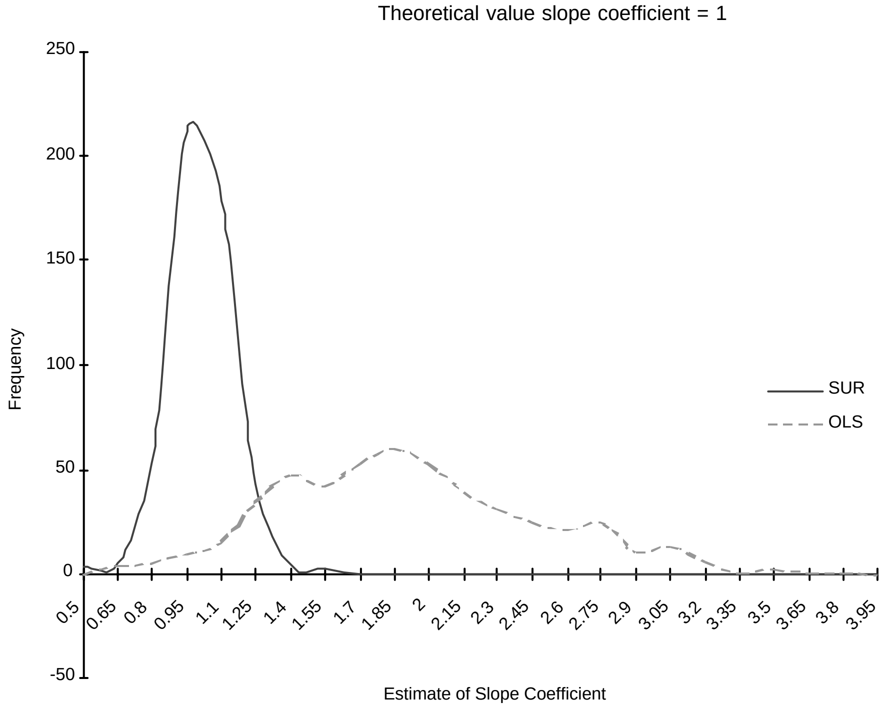

Fig. 4. Bias reduction of simultaneous equation approach based on Monte Carlo simulations. The dotted line represents the distribution of the average slope coefficient of Campbell and Shiller (1991) regressions for the EH based on univariate regressions. The solid line represents the distribution of the slope coefficient based on a SUR pooled regression. The values in excess to one represents the extent of positive bias.

$$b_\kappa = \frac{n\sigma_T^2 - (n+1)\sigma_{(T,t+\Delta t)} + \sigma_{t+\Delta t}^2}{\sigma_{[T-(t+\Delta t)]}^2 + \sigma_T^2 + \sigma_{t+\Delta t}^2 - 2\sigma_{(T,t+\Delta t)}}, \tag{33}$$

where $\sigma_{[T-(t+\Delta t)]}^2$ is the variance of the slope of the term structure of repo spread. Type A regressions are clearly more prone to measurement error. Furthermore, the differential in the impact of noisy data on the two types of regression increases with the tenor lag between the short- and long-term repo spreads.

Since no closed form solution for the size of the distortion is available for simultaneous equations, we use a Monte Carlo approach to estimate the potential distortions in the SUR regression coefficients for different, arbitrarily specified, levels of measurement error. We consider the same auxiliary model used to estimate the small sample bias to describe the dynamics of what we assume to be the true repo spread $L^i(t, t+\Delta t)$ and $L^i(t, T)$, here we also explicitly account for measurement error.

We analyze four different scenarios, indexed by increasing levels of noise. To emphasize the effect of measurement error, we set $\sigma_{(T,t+\Delta t)} = 0$. Scenario 0 is characterized by no noise, and it is used as a control variable. The other scenarios differ from the base case in terms of the proportion $\omega_i = \sigma_{\varepsilon^i}/\sigma_{L^i}$ of noise with respect to the standard deviation of the innovations in the (simulated) true repo rate process. We set, respectively, $\omega_1 = 0.2$, $\omega_2 = 0.4$, and $\omega_3 = 0.6$.

We run 200 Monte Carlo experiments for each regression type with measurement error of different size. In each scenario, we estimate SUR regression coefficients and calculate the difference between the slope coefficients. We use the same set of random numbers across the four scenarios; being the coefficient estimates correlated across the different scenarios, the bias estimates converge quite quickly.

Table 11 reports the Monte Carlo results. We mark with an asterisk the level of bias that would explain a spurious rejection of the expectations hypothesis at a 5% confidence level when the measurement error is not taken into account.

The results from the Monte Carlo experiment confirm that measurement error can significantly affect the estimated regression coefficients, especially in type A regressions. The larger the observation error, with respect to the innovations in the underlying true repo spread, the larger the size of the bias. According to the simulations, if the measurement error is 40% and 60% of the standard deviation of the underlying true repo spread process, in two cases for both types of regression there would be spurious rejection of the null hypothesis.

But how realistic is it to postulate levels of measurement error equal to 40% or even 60% of the innovation term? Can our data set be affected by such large measurement errors? In order to obtain an empirical measure of the level of noise in the data, we derive moment restrictions on the observed (and noisy) repo spread process according to the specification adopted in the Monte Carlo simulation. We identify and estimate σ_L^2 (variance of the true repo process) and σ_ε^2 (variance of measurement error) and then calculate the empirical value of $\omega = \sigma_\varepsilon/\sigma_L$.

Table 11
Estimation of bias due to measurement error
In this table we report estimates of the bias that could be generated by observation error. Let σ_ε^2 be the standard deviation of the innovation in the observation error and let σ_L^2 be the standard deviation of the measurement error. For different values of $\sigma_\varepsilon^2/\sigma_L^2$, we report the values of the bias. The values marked with an asterisk are the values for which the rejection of the spread expectation hypothesis at the 5% confidence level would be explained by measurement error.

Regression type	$\sigma_{\varepsilon^i}/\sigma_{L^i} = 0.20$	$\sigma_{\varepsilon^i}/\sigma_{L^i} = 0.40$	$\sigma_{\varepsilon^i}/\sigma_{L^i} = 0.60$
A1	0.117036	0.445307	0.844678
A2	0.18798	0.686143*	1.327446*
A3	0.132701	0.444155	0.735419
A4	0.092764	0.327326	0.599935
B1	0.033036	0.112156*	0.212638*
B2	H_0 accepted	H_0 accepted	H_0 accepted
B3	0.070792	0.220022	0.362591
B4	0.058708	0.140521	0.254217

*Denotes the level of bias that would explain a spurious rejection of the expectations hypothesis at a 5% confidence level when the measurement error is not taken into account.

Straightforward calculations show that, given Eqs. (26), (29), and (32), the unconditional population moments of the observed spread satisfy the following restrictions:

$$\mathrm{E}_0\left[\hat{L}_t - \frac{\mu}{1-\rho}\right] = 0, \quad \mathrm{E}_0[\hat{L}_{t-1}(\hat{L}_t - \mu - \rho\hat{L}_{t-1})] = 0, \tag{34}$$

$$\mathrm{Var}_0[\Delta\hat{L}_t] = 2\sigma_\varepsilon^2 + \sigma_L^2\left(\frac{2}{1+\rho}\right), \tag{35}$$

$$\mathrm{Cov}_0[\Delta\hat{L}_t, \Delta\hat{L}_{t-1}] = -\sigma_\varepsilon^2 - \sigma_L^2\left(\frac{1-\rho}{1+\rho}\right). \tag{36}$$

We have four independent orthogonality conditions and four parameters. We estimate the model for the one month and one week repo spread processes, both on an individual bond basis and jointly on all specials, imposing the same data generating process. The average values[17] for ω are respectively 4.83% for the one week repo spread process and 12.86% for the one month process. These results suggest that the relative importance of measurement errors is such that values for ω such as 40% or 60% overstate the problem. We therefore do not have evidence that the rejection of the expectations hypothesis has been caused by noise in the data. We also test if the rejection can be due to a time-varying Jensen effect coming from the convexity of bond prices. The answer is negative.

6. The liquidity risk premium

The strong rejection of the spread expectation hypothesis can be interpreted using behavioural arguments based on the limited rationalities of market agents, the segmentation between the short-end and the long-end of the repo market or preferred habitats. Alternatively, the deviation may be due to the existence of a time-varying risk premium. The economic reasons supporting the existence of a liquidity risk premium in this market can be twofold.

First, the risk profile of the book of a typical repo trader is far from being well diversified. Generally, repo traders are specialized and their positions focus on a limited numbers of bonds.[18] Thus, their marginal utilities and trading decisions may be affected by the exposure to idiosyncratic liquidity shocks that may lead to a liquidity risk premium. Furthermore, squeezes in the repo market on bonds close to the cheapest-to-deliver can be correlated. This may reduce the possibility to diversify away liquidity shocks.

[17] In the interest of brevity we do not report here the full regression results.

[18] Typically, repo desks specialize on different segments of the term structure as each trader cannot follow hundred different bonds at the same time.

Second, one may argue that there is an economic rationale supporting a correlation between special repo spreads and market-wide shocks.

(1) When large market shocks induce portfolio reallocations toward safe-heaven assets, such as G7 Government bonds, benchmark bonds are the preferred assets held by institutional investors. Thus, they tend to become temporarily expensive in the cash market. As long as the repo spread fully reflects the temporary high convenience yield of these assets, one may anticipate a correlation between large market shocks and the repo spread.

(2) Although repo contracts are fully collateralized at the time the contract is initiated, sharp changes in market conditions can lead to remarking-to-market the collateral and to margin calls. This issue is particularly acute for convergence trades.[19] If the prices between the two securities further diverge and the repo spread widens, the trader is called on both legs of the trade so that its counterpart is exposed to a risk of default. It is casual evidence that large market shocks are correlated with this form of default risk. This problem played an important role in the liquidity distress suffered by Long Term Capital Management (LTCM) and several other hedge funds in the Fall 1998. In the case of LTCM, the cost of liquidity distress materialized in a substantial injection of capital and collateral by a syndicate of banks that took over its positions.

In order to test the hypothesis of the existence of a time-varying risk premium, we estimate a Garch-in-mean (M-Garch henceforth) model and test (1) if the conditional volatility of the repo spread is a priced risk factor in the repo market and (2) whether the liquidity risk premium is time-varying. We consider the following specification, which has been studied by Engle et al. (1987) for different purposes, for type A regressions:

$$\begin{aligned} &L^i(t+\Delta t, T) - L^i(t,T) \\ &\quad = \alpha^i \sigma_t^2(\varepsilon_{2,t+1}) + \beta^i \left[\frac{\Delta t}{T-(t+\Delta t)}\right][L^i(t,T) - L^i(t,t+\Delta t)] + \varepsilon^i_{1,t+1}, \end{aligned} \tag{37}$$

$$L^i(t+\Delta t, T) = \gamma_1^i + \gamma_2^i L^i(t,T) + \varepsilon^i_{2,t+1}, \tag{38}$$

$$\sigma_t^2(\varepsilon_{2,t+1}) = a^i + b^i \sigma_{t-1}^2(\varepsilon_{2,t}) + c^i(\varepsilon^i_{2,t})^2. \tag{39}$$

The first term on the right hand side of Eq. (38) specifies the time-varying risk premium as a function of the conditional volatility of the repo spread $\sigma_t(\varepsilon_{2,t+1})$. The second term on the right hand side of Eq. (38) is the slope of the term structure of repo spreads as in the first specification of the test of the spread expectation hypothesis. The equivalent specification is used for the test of type B.

We first estimate the model on each of the 43 bonds. The aggregate results are reported in Table 12, Panel A. The evidence of an existence of a risk premium is more favorable in the time-varying specification. The average estimated values of α are all negative, as expected if the intercept were to capture a risk premium. The price

[19] For convergence trades we mean trades in which the investor takes the view that the current price difference between two securities is larger than its equilibrium value.

Table 12

M-Garch (individual regressions)

This table reports the results of M-Garch regressions. In Panel A, we present results from individual bond M-Garch regressions. The average of the regression coefficients is computed across the 43 bonds. In Panel B, we present results from M-Garch regressions for the pooled dataset. In this panel, the regression coefficient $[\alpha, \beta, \gamma, a, b, c]$ are restricted to be the same across all different bonds. A1–B4 are the eight specifications of the expectation hypothesis, depending on the tenors of the contracts. The values of t-stat H_0:$\alpha = 0$ are the t statistics for a test of the null hypohesis $\alpha = 0$, and similarly for $\beta = 0$ and 1. The standard deviations are corrected for heteroskedasticity and autocorrelation using Newey-West.

Type A regression:

$$pcL^i(t+\Delta t, T) - L^i(t,T) = \alpha^i \sigma_t(\varepsilon_{2,t+1}) + \beta^i \left[\frac{\Delta t}{T-(t+\Delta t)}\right][L^i(t,T) - L^i(t, t+\Delta t)] + \varepsilon^i_{1,t+1},$$

$$L^i(t+\Delta t, T) = \gamma_1^i + \gamma_2^i L^i(t,T) + \varepsilon^i_{2,t+1},$$

$$\sigma_t^2(\varepsilon_{2,t+1}) = a + b\sigma^2_{t-1}(\varepsilon_{2,t}) + c(\varepsilon^i_{2,t})^2.$$

Type B regression:

$$\sum_{i=1}^{n-1}\left(1-\frac{s}{n}\right)\Delta^s L^i = \alpha^i \sigma_t(\varepsilon_{2,t+1}) + \beta^i[L^i(t,T) - L^i(t,t+\Delta t)] + \varepsilon^i_{1,t+1},$$

$$L^i(t+\Delta t, T) = \gamma_1^i + \gamma_2^i L^i(t,T) + \varepsilon^i_{2,t+1},$$

$$\sigma_t^2(\varepsilon_{2,t+1}) = a + b\sigma^2_{t-1}(\varepsilon_{2,t}) + c(\varepsilon^i_{2,t})^2.$$

Term Repo Spreads Combinations

Regression	*Short-term tenor*	*Long-term tenor*
A1 and B1	$\Delta t = 1$ week	$T = 1$ month
A2 and B2	$\Delta t = 1$ week	$T = 3$ month
A3 and B3	$\Delta t = 1$ month	$T = 2$ month
A4 and B4	$\Delta t = 1$ month	$T = 3$ month

Panel A: *Individual regressions*

	Avg α	Avg β	Avg R^2	Rej. rate $\alpha = 0$	Rej. rate $\beta = 0$	Rej. rate $\beta = 1$
A1	− 10.79	− 0.70	17%	11%	68%	86%
A2	− 10.79	− 2.02	14%	27%	75%	86%
A3	− 62.04	− 0.53	22%	32%	68%	95%
A4	− 46.43	− 0.73	22%	30%	59%	91%
B1	− 13.17	0.53	35%	20%	55%	84%
B2	− 26.55	0.54	51%	50%	70%	75%
B3	− 31.02	0.24	19%	32%	32%	95%
B4	− 37.91	0.34	29%	45%	39%	91%
Average	− 29.86	− 0.29	26%	31%	58%	88%

Table 12 (*continued*)

Panel B: *Garch estimation* Regression type	α (std error)	β (std error)	R^2	$H_0{:}\alpha = 0$ p-value	$H_0{:}\beta = 0$ p-value	$H_0{:}\beta = 1$ p-value
Type A regression						
A1	−0.156 (0.026)	0.102 (0.084)	0.03%	0.00	0.22	0.00
A2	0.019 (0.021)	0.421 (0.140)	0.02%	0.34	0.00	0.00
A3	−0.383 (0.104)	−0.124 (0.127)	13%	0.00	0.33	0.00
A4	−0.258 (0.109)	−0.130 (0.129)	13%	0.02	0.31	0.00
Type B regression						
B1	−0.060 (0.050)	0.809 (0.045)	30%	0.23	0.00	0.00
B2	0.004 (0.146)	0.921 (0.067)	48%	0.98	0.00	0.24
B3	−0.103 (0.124)	0.492 (0.068)	19%	0.41	0.00	0.00
B4	−0.261 (0.098)	−0.520 (0.053)	23%	0.01	0.00	0.00

of volatility risk is statistically different from zero in 31% of the cases, and for 27% of all bonds the price of volatility risk is strictly positive ($\alpha < 0$). Furthermore, when the intercept is statistically different from zero, its point estimate is negative in more than 87% of cases. This compares with the previous constant intercept specification in which the signs of the intercept were roughly equally split between negative and positive values and the standard error of the estimate were substantially higher.

The result is particularly strong when we aggregate the data set and run the test on the panel of bonds that traded special. This approach has the benefit of reducing substantially any potential small sample bias in a way that is not model-specific. As a matter of fact, the size of correction for small sample bias obtained using a Monte Carlo approach depends on the assumptions made on the data generating process. Table 12, Panel B reports the results. We find that in three out of four tests of type A and in three out of four tests of type B the value of α is negative and in three out of four cases, in tests of type A the null hypothesis of the absence of a liquidity risk premium is statistically rejected at the 5% confidence level. Moreover, the R^2 of the regressions increase to an overall average of 16% and 30% for tests of type B and the β slope coefficient increase toward one.

Since the special repo spread is usually considered a proxy for liquidity, this supports the argument that liquidity risk is priced and that it should be taken into account by value-at-risk measures or for the construction of risk-adjusted performance measures. Statistical significance aside, what is the economic size of the risk premium? Given the estimated parameters of the Garch-in-mean, the time-average of the liquidity risk premium, i.e., $(1/T)\sum_t \alpha^i \sigma_t^2(\varepsilon_{2,t+1})$, is equal to $8bp$,

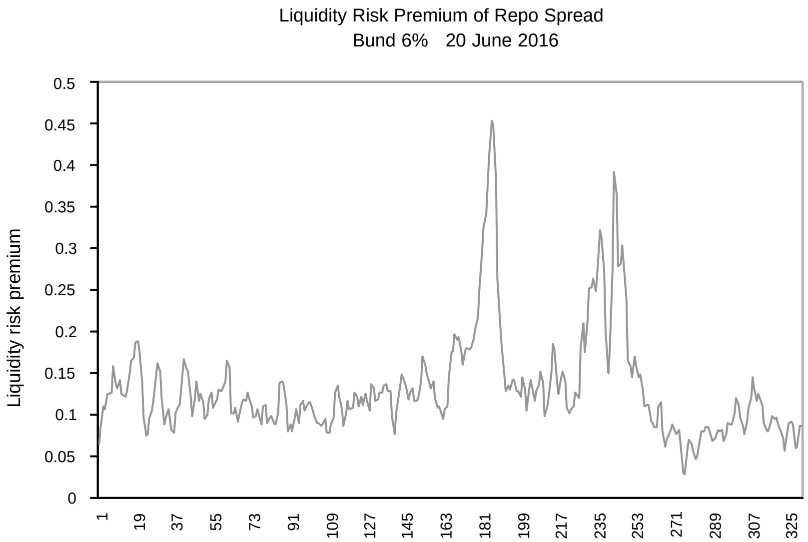

Fig. 5. Time-varying liquidity risk premium, embedded in the repo spread of the Bund 6% 2016. The estimate is obtained by estimating $\alpha^i \sigma_t(\varepsilon_{2,t+1})$ in the following Garch in mean:

$$L^i(t+\Delta t, T) - L^i(t,T) = \alpha^i \sigma_t(\varepsilon_{2,t+1}) + \beta^i \left[\frac{\Delta t}{T-(t+\Delta t)}\right][L^i(t,T) - L^i(t, t+\Delta t)] + \varepsilon^i_{1,t+1},$$

$$L^i(t+\Delta t, T) = \gamma^i_1 + \gamma^i_2 L^i(t,T) + \varepsilon^i_{2,t+1},$$

$$\sigma^2_t(\varepsilon_{2,t+1}) = a^i + b^i \sigma^2_{t-1}(\varepsilon_{2,t}) + c^i(\varepsilon^i_{2,t})^2.$$

The risk premium in negative, consistently with the fact that the repo spread $L^i < 0$ and the hypothesis that the long-term repo spreads embed a liquidity risk premium. For convenience we plot $-\alpha^i \sigma_t(\varepsilon_{2,t+1})$, which is positive.

which constitutes almost 25% of the average repo spread. Moreover, Fig. 5 shows that the liquidity risk premium is highly time-varying. It ranges from few basis points to about 45*bp*. The extent of the such time variation can explain the sound rejection of the spread expectation hypothesis.

7. Interpretation and conclusions

Both individual and panel data regressions strongly reject the EH in its general formulation. Moreover, the analysis based on a Garch-in-Mean approach suggests that this risk premium is economically large, time-varying and correlated to the conditional volatility of the spread. What is the economic explanation for such a result? How different is a rejection of the EH in the special repo market from the general level of interest rates?

It is very reasonable to expect that the general *level* of the interest rates is subject to market-wide shocks. However, it is difficult to imagine anything more idiosyncratic than special repo *spreads*. Thus, why should shocks to repo spreads shocks be priced by the market and why should the risk premium embedded in the term structure of repo spread be time-varying?

A typical repo trader is highly specialized. The relatively small number of securities a trader actively trades limits the possibility to diversify liquidity risk across a large cross-section of bonds. Second, in situations of large market shocks, typically characterized by large portfolio reallocations to benchmark bonds (“flight-to-quality”), the correlation between repo spreads of these bonds increases. This reduces even more the possibility to diversify away the liquidity risk. Third, repo transactions are collateralized, to minimize the credit risk arising from a default of the counterpart. Nonetheless certain embedded risks remain. Some of these risks include liquidity risk. There are two types of liquidity risks: refinancing difficulties and counterpart default. The first arises when it becomes difficult to roll-over maturing positions. This can become particularly acute when the financial institution has a term structure of repo-debt positions too unbalanced toward the short-end. In case of large market swings, a dry up of liquidity may cause a situation of liquidity distress. The second type of liquidity risk comes from the default of the counterpart in a market scenario of increased illiquidity, generally following large market wide shocks. If liquidity has dried up, the liquidation of the collateral may be achieved only at a discount.

Recent evidence on convergence trades offers a good example of the credit and liquidity risks embedded in strategies designed to take advantage of the mean-reversion in yields or repo spreads between two securities, such as corporate bonds versus treasuries or general collateral bonds versus a special bond. When the convergence does work, the related repo and reverse repo liquidity exposure and related margin calls offset. However, if the spread widens with a contemporaneous market wide shock in the level of interest rates, the spread position can cause margin calls on both the repo and reverse repo transaction. The practice of remarking-to-market the value of the collateral posted, in order to reduce the credit risk can increase the liquidity risk. It is well known that these situations have affected several hedge funds who have suffered liquidity distress during the summer of 1998.

When we estimate and test if the risk premium is correlated to the time-varying conditional volatility of the special repo spread, we find supporting evidence for this explanation.

7.1. Conclusions

In this paper we have analyzed the collateral value of Treasury bonds. In the repo market it is common to observe that bonds with otherwise similar cash flow payments may trade at large spreads.

In order to learn more about the empirical properties of this important market, we study the term structure of repo spreads in the framework of the expectations hypothesis, allowing for a constant risk premium. We ask two questions: What is the extent to which the repo market discount the future collateral value (specialness) of Treasury bonds? and is liquidity risk priced in the repo market? The first questions is of interest since short positions in the cash market can be financed either by rolling over short-term repo contracts or by long-term repos. The second question is of interest especially in the aftermath of the Fall 1998 crises, in which several hedged funds, highly leveraged in the repo market, suffered conditions of liquidity distress.

We estimate and test two sets of restrictions based on the expectations hypothesis, using both individual bond regressions and a simultaneous regression approach. Then, we estimate and test a M-Garch model to explore the existence (and characteristics) of a time-varying liquidity risk premium.

We learn the following from the empirical analysis of the repo spreads:

1. The EH is strongly rejected by the data. The slope coefficient of both individual bonds and aggregate bonds regressions is significantly lower than one. In one of the two specifications, the deviations are so large that, in three cases out of four, the current slope of the term structure of special repo spreads is negatively correlated with future changes in long term special repo spreads. High relative levels of long term repo rates overestimate the extent to which a bond will remain special and/or the extent of the specialness.

2. Current long term repo spreads poorly anticipate future convenience yields generated by long term repo rates: the R^2 of regressions of the first type is extremely low.

Moreover, when we run tests of robustness regarding the previous results, we also learn that the following arguments, which are sometimes suggested to explain the deviations from the expectations hypothesis of interest rates, do not apply to the case of the term structure of special repo spreads.

3. Deviations from the expectations hypothesis cannot be due simply to small sample bias. We address this issue both using a Monte Carlo experiment and by using a simultaneous regression approach.

4. The rejection of the expectations hypothesis is not due to the effect of measurement error. Using a Monte Carlo approach we find that the size of the measurement error must be too large to generate a bias of size consistent with the extent of the deviation from the spread expectation hypothesis.

5. We show that the economic significance of the risk premium (deviations from the expectations hypothesis) is large enough to exclude an explanation based on a lack of an enforcement mechanism. We use a M-Garch approach and find that the deviations are indeed due to a time-varying risk premium, as opposed to market segmentation or pure overreaction, explained by the conditional volatility of the special repo spread. The component of the special repo spread that constitutes the time-varying part of the risk premium is on average 25% of the total spread and it can be as high as 45*bp* basis points for a significant number of days.

The time-varying risk premium may be related to the fact that although repo transactions are collateralized, so to minimize the credit risk arising from a default of the counterpart, certain embedded risks, such as liquidity risk, remain. The experience of financial institutions financing convergence trades in the Summer 1998 using repos gave very visible examples of the impact of margin calls in situations of liquidity contraction.

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