Term Structure Models with Differences in Beliefs

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Term Structure Puzzles

Several properties of bond prices are difficult to reconcile with traditional models:

1. **Predictability:** violations of the EH: (a) Fama and Bliss (1987), Cochrane and Piazzesi (2005) for excess returns; (b) Campbell and Schiller (1991) neg regression coefficients; (c) also predictability in high-freq [Mueller, Vedolin, Yen (2011)]
2. **Volatility anomalies:** (a) long term bond yields appear too volatile to accord with traditional rational expectations models; (b) the term structure of volatility is itself a complex object. [Shiller (1979), Piazzesi (2005), and Piazzesi and Schneider (2006)] .
3. **Statistical properties of bonds:** (a) essentially affine models cannot simultaneously match conditional first and second moments of yields; (b) traditional affine models imply perfect spanning which is strongly rejected in the data. [Duffee(2002), Duffee(2008), Joslin, Priebsch, and Singleton (2011)] .
4. **Unspanned stochastic volatility:** Bonds do not span interest rate derivatives [Collin-Dufresne and Goldstein (2002a), Heidari and Wu (2003a), Li and Zhao (2006)].

Our Contribution

We investigate empirically a specific economic channel:

- The properties of the SDF change when one moves from single agent to multiple agent economy; In particular:
  1. **Bond Risk Premia**
  2. **Bond Volatility**
  3. **Bond Trading Activity**

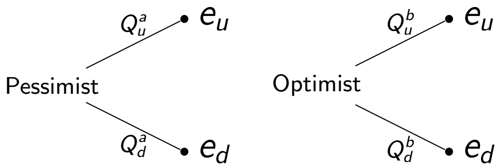
Builds on earlier work in the Literature on Differences in Beliefs:

- Theory: Harris and Raviv (1993), Detemple and Murthy (1994) and Zapatero (1998), Buraschi and Jiltsov (2006) Scheinkman and Xiong (2003), Dumas, Kurshev and Uppal (2009), Bhamra and Uppal (2009).

- Empirical: [Equity] Diether, Malloy, and Scherbina (2002); [Correlation Risk Premium] Buraschi, Trojani and Vedolin (2009), [FX] Buraschi, Beber, Breeden (2010), [Credit] Buraschi, Trojani, Vedolin (2009). [Bonds]: Xiong and Yan (2010) who study speculative trading regarding the central banks inflation target.

Heterogeneous Belief Structure

Consider an economy with two agents with different subjective conditional probability measures dQ_t^a and dQ_t^b.



$$Q_u^b > Q_u^a$$

$$Q_d^b < Q_d^a$$

If absolute continuous, a convenient way to summarize difference in subjective measures (disagreement) is the Radon-Nikodym derivative process $\eta = \frac{dQ^b}{dQ^a}$;

if X_t be an $\Im_t$-measurable random variable then

$$E^b(X_T|\Im_t) = E^a\left(\frac{\eta_T}{\eta_t}X_T|\Im_t\right).$$

The Homogeneous Benchmark Economy

- Assume $u'(c_t) = e^{-\rho t} c_t^{-\gamma}$ and

$$\begin{aligned} dD_t/D_t &= \beta' \mathbf{g}_t dt + \sigma_D d\mathbf{W}_t^D \\ d\mathbf{g}_t &= -\kappa_g(\mathbf{g}_t - \theta)dt + \sigma_g d\mathbf{W}_t^g. \end{aligned}$$

- Common beliefs: $d\mathcal{M}^*/\mathcal{M}^* = -r_t dt - \kappa' dW_t^*$, since $\mathcal{M}_t^* = u'(D_t)$:

$$r_t = \delta + \gamma\beta'\mathbf{g}_t - \frac{1}{2}\gamma(1+\gamma)\sigma_D^2.$$

- If growth rates are constant, so are interest rates and the term structure is flat.

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- If growth rates are constant, so are interest rates and the term structure is flat.
- When g_t is stochastic:

$$B_t^{(T-t)} = E_t^* \left[\frac{M_T^*}{M_t^*}\right] = \exp\left[A_h(t,T) + G_h(t,T)\mathbf{g}_t\right]$$

  Excess returns:

$$rx_{t,t+dt}^{(T)} = -\gamma G(t,T)\sigma_D\sigma_g E\left(dW_t^D dW_t^g\right).$$

- Very tight connection between volatility and risk compensation

Introduce Heterogeneity

- Growth rates are unobservable;
- Assume two factors: $g_t = [g_t^1, g_t^2]$

$$dg_t^i = -\kappa_{g^i}^i (g_t^i - \theta_g^i) dt + \sigma_g^i d\hat{W}_t^{g^i}.$$

- suppose agents agree on σ_g^i and θ_g^i but disagree on $\kappa_{g^i}^i$.
- Since g_t is unobservable it is (a) hard to agree on it's dynamics; and (b) the relative importance of long-run versus business cycle components [see Hansen, Heaton and Li (2008); Pastor and Stambaugh (2006)].
- Difference in beliefs summarized by $\eta_t = \frac{d\mathcal{P}_t^b}{d\mathcal{P}^b}$, so that

$$E^b(X_T | \Im_t) = E^a \left(\frac{\eta_T}{\eta_t} X_T | \Im_t \right).$$

Immediate Implications

- **Example**. (Xiong and Yan (2010)): Agents have ex-ante incentives to trade with each others. If, $u(c^i) = e^{-\rho(T-t)} \ln c_t^i$, agents trade until their wealth ratio is equal to

$$W_T^b / W_T^a = \eta_T$$

- Complete markets imply perfect risk sharing only if $dQ_t^a = dQ_t^b$.
- Furthermore, in equilibrium

$$B_t^{(T-t)} = \frac{1}{1+\eta_t} B_t^{(T-t),a} + \frac{\eta_t}{1+\eta_t} B_t^{(T-t),b}$$

- Even if η_t were constant, bond prices in the heterogeneous economy would not be affine!

Sketch Consider tradable asset with terminal payoff B_T. In equilibrium, agents must agree on its value. Under logarithmic preferences, this requires $E_t^b(\frac{c_t^b}{c_T^b} B_T) = E_t^a(\frac{c_t^a}{c_T^a} B_T)$. From Merton, $c_t^i = \rho W_t^i$, so that $E_t^b(\frac{W_t^b}{W_T^b} B_T) = E_t^a(\frac{W_t^a}{W_T^a} B_T)$. Define $\frac{W_t^b}{W_T^b} B_T = \tilde{B}_T$, then $E_t^b(\tilde{B}_T) = E_t^a\left[(\frac{W_T^b / W_T^a}{W_t^b / W_t^a})\tilde{B}_T\right]$, so that $\frac{\eta_T}{\eta_t} = \frac{W_T^b / W_T^a}{W_t^b / W_t^a}$.

A General Model

- Allow for Learning. Two types of signals in the literature:
- **First-order signals**: a process S^i_t whose drift is correlated with the *conditional first moments* of $\hat{g}_t$ or $E_t[dD_t]$ [Detemple (1986), Veronesi (2000), Buraschi and Jiltsov, 2006].

$$dg_t = -\kappa_g(g_t - \theta)dt + \sigma_g dW^g_t,$$
$$dS^i_t = \left(\phi g_t + (1-\phi)\varepsilon^i_t\right) dt + \sigma^i_s dW^{S^i}_t.$$

- **Second-order signals**: a process S^j_t whose Brownians are correlated with the *second moments* of $\hat{g}_t$ or $\langle dD_t \rangle^2$ [Scheinman and Xiong, 2003, Dumas, Krushev and Uppal, 2009, Xiong and Yan, 2010].

$$dg_t = -\kappa_g(g_t - \theta)dt + \sigma_g dW^g_t,$$
$$dS^j_t = \sigma^j_s \phi dW^g_t + \sigma^j_s \sqrt{1-\phi^2} dW^{S^j}_t.$$

- Filtering using first versus second order filtering has important pricing implications.

Learning

- Conditional on priors agents are rational.
- **State-space representation**: $X_t = [\log D_t \ , S_t^1 \ , S_t^2]'$ and $\mu_t = [g_t^1 \ , g_t^2 \ , \varepsilon]'$,

$$dX_t = (A_0 + A_1 \mu_t)\, dt + B dW_t^X$$
$$d\mu_t = (a_0 + a_1 \mu_t)\, dt + b dW_t^{\mu},$$

- **Lemma**. (Beliefs) The posteriors solve:

$$dm_t^n = (a_0 + a_1 m_t) dt + \upsilon_t A_1' B^{-1} d\hat{W}_t^{X,n}$$
$$\dot{\upsilon}_t = a_1 v_t + \upsilon_t a_1' + bb' - \upsilon_t A_1' (BB')^{-1} A_1 \upsilon_t$$

where

$$d\hat{W}_t^X = B^{-1}[dX_t - (A_0 + A_1 \mu_t)\, dt].$$

- Agents have different **subjective** Brownians

$$d\hat{W}_t^{X,b} = d\hat{W}_t^{X,a} + B^{-1} A_1 (m_t^a - m_t^b) dt.$$

- Define $\Psi_t \equiv B^{-1} A_1 (m_t^a - m_t^b)$ as the standardized difference in beliefs.

Equilibrium

- Problem $\max_{\{c^i, \mathcal{M}^i\}} E_o^i \int_0^\infty \varrho_t u(c_t^i) dt$

- s.t. $E_0^i \int_0^\infty \mathcal{M}_t^i \left[c_t^i - e_t^i \right] dt \leq 0$; $\sum_i c_t^i = D_t$ for $\forall t$.

- Solution:

$$\eta_t = \frac{\alpha_b}{\alpha_a} \frac{u_a'(c_t)}{u_b'(c_t)} = \frac{\mathcal{M}_t^a}{\mathcal{M}_t^b}.$$

$$d\eta_t / \eta_t = -\Psi_t d\hat{W}_t^{X,a}$$

- Disagreement Ψ_t affects the $\mathcal{M}_t$ process, thus directly bond prices.

Bond Market Implications

- Stochastic discount factor

$$
\begin{aligned}
d\mathcal{M}_t^*/\mathcal{M}_t^* &= -r_f(t,\Psi_t)dt - \kappa^*(t,\Psi_t)d\hat{W}_t^{X,*} \\
B_t^{(T-t)} &= E_t^*(\mathcal{M}_T^*/\mathcal{M}_t^*)
\end{aligned}
$$

$$
\mathcal{M}_t^* = \left[\underbrace{\varrho_t D_t^{-\gamma}}_{\text{Lucas term}}\right]\left(\underbrace{1+(\alpha\eta_t)^{1/\gamma}}_{\text{Disagreement}}\right)^{\gamma}
$$

Interest Rate

- From the drift of $d\mathcal{M}_t^*$: risk free rate

$$r_f = \rho + \gamma\beta' \underbrace{\left(\omega_a(t)\hat{g}_t^a + \omega_b(t)\hat{g}_t^b\right)}_{\text{Consensus Aggregation Bias}} - \underbrace{\frac{1}{2}\gamma(\gamma+1)\sigma_D^2}_{\text{Precautionary Savings}} + \underbrace{\frac{\gamma-1}{2\gamma}\omega_a(t)\omega_b(t)\Psi_t{}'\Psi_t}_{\text{Differences in Beliefs}},$$

where $\omega_i(t) = c_t^i / D_t$ is investor's i total consumption share.

- Standard Vasicek economy: $r_f = \rho + \gamma\beta'\hat{g}_t - \frac{1}{2}\gamma(\gamma+1)\sigma_D^2$.

- When $\gamma > 1$ interest rates are increasing in Ψ_t.

Risk Premia

- Vasicek: only priced shocks are $dW_D(t)$ and since dD_t/D_t is homoskedastic, the price of risk is constant.
- Here, more interestingly, the dynamics of $d\mathcal{M}_t^*$ also depend on $d\eta_t$.
  - **Relative consumption is stochastic:** Optimistic (pessimistic) investors consume more (less) in states of high aggregate cash flows, at a lower (higher) marginal utility, because they perceive those states as more (less) likely. This also implies that the consumption volatility of the optimist is higher than the pessimist.

$$\kappa_a'(t) = \gamma\sigma_D + \omega_b(t)\Psi_t \text{ and } \kappa_b'(t) = \gamma\sigma_D - \omega_a(t)\Psi_t.$$

  - Or ...

$$\mathcal{M}_t^i = \tilde{\mathcal{M}}_t^i \times \omega_i(t)^{-\gamma}, \text{ with } \quad \tilde{\mathcal{M}}_t^i \equiv \frac{1}{\alpha_i}\varrho_t D_t^{-\gamma}$$

- Risk does not cancel out at the level of representative agent:

Disagreement drives price of risk

- **Theorem** The price of risk is affected directly by disagreement:

$$\kappa_g^*(t) = \gamma\sigma_D + \omega_b(t)\psi_g(t) \qquad \kappa_s^*(t) = \omega_b(t)\psi_s(t),$$

  which implies that bond excess returns explicitly depend on the dynamics of Ψ_t.

- This channels are absent in Jouini and Napp (2010): constant beliefs and no learning; or in Xiong and Yan (2010) with $\gamma = 1$. Different than in the Vasicek benchmark economy the price of risk is time varying.

... AND OF SIGNALS

- **Corollary** [Dimensionality of the State Space] Suppose that agents use $N + M$ signals for inference. Let N be the number of signals correlated with *expected* innovations (first-order signals) and M being the number of signals correlated with *unexpected* innovations of the fundamentals (second-order signals). Then, there are $1 + N$ priced state variables.

- Disagreements on first-order signals, in addition to disagreement on fundamentals, are priced. Second order signals are not priced instantaneously. Signals that affect the conditional first moment of the growth rates enter directly as state variables in agents' belief-dependent optimal consumption plans.
- Thus, this channel is absent in the economies studied in Dumas, Krushev, Uppal (2010) where $\kappa_s^*(t) = 0$.
- **Intuition**: Second order filtering requires no change of measure; thus, no risk sharing on information flow.

The Term Structure of Bond Prices

- **Theorem** [The Term Structure of Bond Prices]
- Exponential linear-quadratic solution in m_D and Ψ. Bond prices are equal to the product of two deterministic functions:
- 1. The first is exponentially affine in the posterior growth rate of the endowment;
  2. the second is exponentially quadratic in the level of differences in beliefs:

$$B(t,T) = \varrho_{T-t} F_{m_D^a}(m_D^a, t, T; -\gamma) G(t, T, -\gamma; \psi_g; \psi_s),$$

$$F_{m_D^a}(m_D^a, \tau, \epsilon) = \exp(A_{m_D^a}(\epsilon, \tau) + B_{m_D^a}(\epsilon, \tau) m_D^a),$$

$$F_{\psi_g, \psi_S} = \exp(A_\Psi(\tau) + B_\Psi(\tau)\Psi_t + \Psi_t C(\tau) \Psi_t').$$

Testable Implications

Hypothesis 1: Expected Returns

$$rx^{\tau}_{t,t+dt} = \underbrace{-\gamma B_{m^a_D}(\tau)\sigma_D\sigma_g E^*\left(d\hat{W}^D_t d\hat{W}^g_t\right)}_{\text{Learning}} + \underbrace{\omega_b(t)\psi_g(t)\sigma^T_{B,D}(t)}_{DiB \text{ fundamentals}} + \underbrace{\omega_b(t)\psi_s(t)\sigma^T_{B,S}(t)}_{DiB \text{ signals}}$$

1. First term homoskedastic.
2. Second term: disagreement on fundamentals (on g_t and potentially π_t)
3. Third term: disagreement on first-order signals (which we proxy using with disagreement about future bond yields).

Testable Implications

Hypothesis 2: Disagreement about (future) prices

- By no-arbitrage all agents must agree on $B(t, T)$ a time t: $B^a(t, T) = B^b(t, T)$.
- But for $t < \tau < T$ disagreement can extend to future bond prices: $E_t^a[B(\tau, T)] \neq E_t^b[B(\tau, T)]$.
- Dispersion in price forecasts is due to $\eta_t(\psi_t^g, \psi_t^S)$ which depends on disagreement about both fundamentals and signals: $E_t^a[B(\tau, T)] = E_t^b\left[\frac{\eta_\tau}{\eta_t} B(\tau, T)\right]$.

Testable Implications

Hypothesis 3: Stochastic Volatility

- Disagreement generate endogenous stochastic volatility of bond yields even if the endowment process is homoskedastic.
- When $\gamma > 1$, changes in disagreement directly affect both the level and slope of the term structure of volatility of (changes in) bond yields:

$$\sigma_{B,D}^{T}(t) = \alpha_{0,D}(\tau) + \alpha_{1,D}(\tau)\psi_g + \alpha_{2,D}(\tau)\psi_s$$
$$\sigma_{B,S}^{T}(t) = \alpha_{0,S}(\tau) + \alpha_{1,S}(\tau)\psi_g + \alpha_{2,S}(\tau)\psi_s$$

Hypothesis 4: Spanning

- Heterogeneous beliefs generate dynamics in bond prices suggestive of unspanned factors.

Hypothesis 5: Trade

- In a heterogeneous agent economy with difference in belief agents risk share by trading state contingent claims; thus, shocks to disagreement induce an increase in trading activity.

Data Description

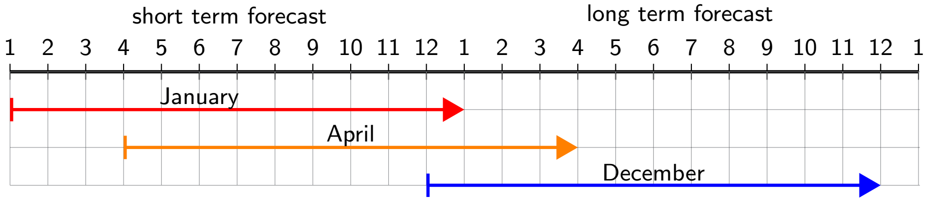
A Unique Dataset on the Formation of Expectations

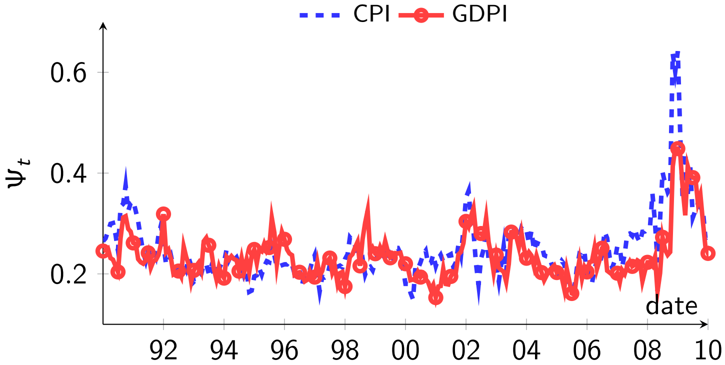
- BlueChip Economic Indicators: Single biggest data set of agent's expectations.
- Unfortunately, digital copies of BCEI are only available since 2007. We sourced the paper archive directly from Wolters Kluwer and digitize the data: it now includes 350,000 data points of **individual** observations on:
  1. Real: Real GDP, Personal Income Expenditure, Unemployment, Industrial Production, Housing Starts, Non-residential Investment, Corporate Profits, Auto and Truck Sales
  2. Nominal: Consumer Price Inflation, GDP Price index.
  3. Monetary: 3 Month bank discount rate; 10Y Treasury Rate.
- We have data on two sets of forecasts for January 1990 - December 2011:
  1. Short-term: an average for the remaining period of the current calendar year.
  2. Long-term: an average for the following year.

Modelling constant maturity DiB

- Important advantages in new dataset:
  1. Monthly frequency (vs. Quarterly frequency of SPF)
  2. Large Cross-section Agents (never less than 40 vs. 9 of SPF)
  3. One agency since 1978
  4. SPF changed questions and/or the forecast horizon

- Can use individual short & long forecast to obtain implied constant maturity forecast with no seasonality (lots of seasonality in SPF, which people filter using X-12 ARIMA - TRAMO/SEATS filters).





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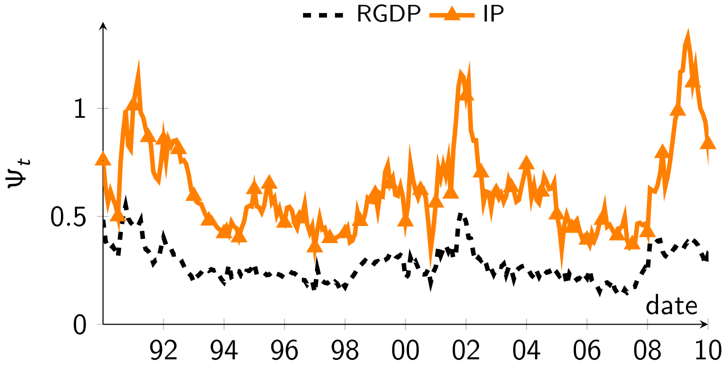

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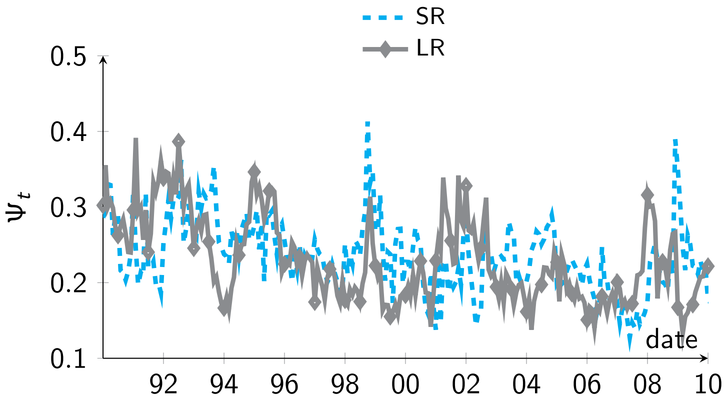

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Hypothesis 3: Return Predictability

$$rx_{t,t+12}^{(5)} = const + \sum_{i=1}^{3} \beta_i \psi_t(\star) + \sum_{i=1}^{4} \gamma_i E_t(\star) + \sum_{i=1}^{11} \phi_i Macro_t(\star) + \delta Slope_t + \varepsilon_{t,t+12},$$

	Ψ^{π}	Ψ^{g}	Ψ^{s}	$Slope_t$	$E(\pi)$	$E(g)$	$E(LR)$	$E(SR)$	$F^1 \rightarrow F^8$	σ^{π}	σ^{g}	$\rho^{\pi,g}$	$\overline{R}^2$
(i)	-0.62	1.85											0.12
	-1.48	3.20											
(ii)	0.03	1.17	2.22										0.35
	-0.07	2.53	4.78										
(iii)	0.07	0.86	2.27	0.61									0.36
	0.15	1.71	4.83	1.15									
(iv)	-0.37	1.69			0.58	-0.01							0.13
	-0.95	1.97			1.18	-0.02							
(v)			2.41				1.07	-1.01					0.31
			4.91				1.34	-1.26					
(vi)	-1.64	1.86							✓	0.00	-0.26	0.45	0.24
	-2.08	2.42								0.01	-0.70	1.31	
(vii)			2.35						✓	-0.17	-0.13	0.29	0.41
			5.85							-0.47	-0.42	1.37	

Hypothesis 3: Price Dispersion

- Hypothesis 2 questions the relative importance of disagreement on fundamentals versus disagreement on future prices. This is important since disagreement on future prices cannot exist in ambiguity models even if disagreement on fundamentals serves as a proxy for Knightian uncertainty:

$$\psi_t^s = -\underset{(-0.49)}{0.06}\,\psi_{\pi,t} + \underset{(1.22)}{0.04}\psi_{g,t} + \underset{(0.01)}{0.02}(\psi_{\pi,t})^2 + \underset{(1.90)}{0.27}(\psi_{g,t})^2 - \underset{(-2.44)}{1.46}\,(\psi_{\pi,t}\cdot\psi_{g,t}) + \varepsilon_t^s$$

$$\overline{R}^2 = 0.14$$

$$\psi_t^s = -\underset{(1.17)}{0.02}E_t[\pi] - \underset{(-0.11)}{0.00}\,E_t[g] + \underset{(-0.65)}{0.00}\,E_t[LR] + \underset{(0.58)}{0.00}E_t[SR] + \varepsilon_t^s$$

$$\overline{R}^2 = 0.07$$

- This results is difficult to reconcile with single agent ambiguity models. Moreover, within the family of heterogeneous beliefs models, we learn that the cross-section of beliefs on asset prices is richer than what can be explained by disagreement on fundamentals alone, highlighting the potential importance of studying learning models with first order signals.

Hypothesis 3: Volatility

$$\sigma^{slope}_{t,t+1} = const + \alpha\sigma^{slope}_{t-1,t} + \sum_{i=1}^{3}\beta_i\psi_t(\star) + \sum_{i=1}^{2}\gamma_i E_t(\star) + \sum_{i=1}^{3}\phi_i Macro_t(\star) + \varepsilon_{t+1},$$

	Ψ^{π}	Ψ^{g}	Ψ^{s}	$E(\pi)$	$E(g)$	$E(LR)$	$E(SR)$	$F^1 \to F^8$	σ^{π}	σ^{g}	$\rho^{\pi,g}$	$\sigma^{slope}_{t-1,t}$	$\overline{R}^2$
(i)	-0.15											0.48	0.35
	-4.05											5.22	
(ii)		-0.05										0.56	0.31
		-1.09										7.63	
(iii)			0.13									0.49	0.34
			3.67									7.51	
(iv)	-0.17	0.01	0.13									0.40	0.39
	-5.24	0.38	3.91									4.69	
(v)	-0.13	0.04	0.13	0.01	0.04	-0.03	0.06					0.39	0.38
	-2.89	0.71	3.90	0.07	0.69	-0.29	0.84					4.51	
(vi)	-0.04	0.02	0.15					✓	-0.04	0.02	-0.01	0.38	0.47
	-0.71	0.40	4.23						-1.00	-0.43	-0.26	4.63	

Hypothesis 4: Spanning

$$\psi_t^i = const + \sum_{i=1}^{5} \beta_i \ PC_t^i + \varepsilon_t^i,$$

	PC^1	PC^2	PC^3	PC^4	PC^5	$\overline{R}^2$
Ψ^π	-0.38	0.09	-0.24	-0.27	0.32	0.37
	-4.70	1.42	-3.16	-2.83	3.02	
Ψ^g	-0.14	0.43	-0.20	0.01	0.19	0.27
	-1.66	6.67	-2.54	0.12	2.45	
Ψ^s	0.20	0.09	-0.18	0.28	-0.11	0.16
	3.06	1.19	-2.15	4.12	-1.36	

Hypothesis 4: Spanning

$$rx_{t,t+12}^{(n)} = const^{(n)} + \sum_{i=1}^{3} \beta_i^{(n)} \psi_{\mathcal{UN}}^{\star} + \varepsilon_{t+12}^{(n)},$$

regressor	$rx^{(2)}$	$rx^{(3)}$	$rx^{(4)}$	$rx^{(5)}$
$\psi_{\mathcal{UN}}^{\pi}$	0.04	0.06	0.10	0.07
	(0.25)	(0.22)	(0.26	0.15)
$\psi_{\mathcal{UN}}^{g}$	0.33	0.56	0.66	0.67
	(2.47)	(2.65)	(2.39)	(2.05)
$\psi_{\mathcal{UN}}^{s}$	0.60	1.20	1.69	2.11
	(5.71)	(5.42)	(5.04)	(4.69)
$\overline{R}^2$	0.28	0.29	0.28	0.27

Hypothesis 4: Spanning

$$Hidden_t = const + \sum_{i=1}^{4} \beta_i \; \psi^{i}_{\mathcal{UN}} + \varepsilon^{i}_{t},$$

	$\psi^{\pi}_{\mathcal{UN}}$	$\psi^{g}_{\mathcal{UN}}$	$\psi^{s}_{\mathcal{UN}}$	$\overline{R}^2$
$Hidden_t$	-0.02	-0.01	0.20	0.03
	(-0.23)	(-0.14)	(2.24)	

Hypothesis 4: Spanning

$$rx_{t,t+12}^{(n)} = const^{(n)} + \sum_{i=1}^{3} \beta_i^{(n)} \psi_{\mathcal{AB}}^{\star} + \varepsilon_{t+12}^{(n)},$$

regressor	$rx^{(2)}$	$rx^{(3)}$	$rx^{(4)}$	$rx^{(5)}$
$\psi_{\mathcal{AB}}^{\pi}$	0.08	0.10	0.22	0.14
	(0.39)	(0.27)	(0.43)	(0.21)
$\psi_{\mathcal{AB}}^{g}$	0.36	0.59	0.62	0.57
	(2.05)	(1.96)	(1.55)	(1.19)
$\psi_{\mathcal{AB}}^{s}$	0.52	1.08	1.51	1.88
	(3.48)	(3.60)	(3.42)	(3.29)
$\overline{R}^2$	0.22	0.22	0.20	0.19

Hypothesis 5: Trade

$$Vol_{t,t+1} = const + \sum_{i=1}^{3} \beta_i \psi_t(\star) + \gamma Vol_{t,t-1} + \varepsilon_{t,t+1}.,$$

regressor	(i)	(ii)	(iii)	(iv)	(v)	(vi)
ψ^{π}	0.71		0.46		0.43	0.26
	(8.88)		(4.07)		(3.83)	(2.89)
ψ^{g}		0.68	0.39		0.40	0.18
		(3.62)	(2.69)		(2.95)	(2.10)
ψ^{s}				-0.22	-0.07	0.00
				(-1.14)	(-0.89)	(0.05)
$Vol_{t,t-1}$						0.51
						(8.98)
$\overline{R}^2$	0.50	0.47	0.60	0.04	0.59	0.70

$Vol_{t,t+1}$ is the log ratio of the number of monthly transaction volumes between primary dealers and secondary customers for Treasury Bills versus coupon paying securities due in more than 6 years but less than or equal to 11 years.

Conclusions

Existence of time-varying bond risk premia is one of the most interesting and challenging topic in fixed income. The weak empirical link between observable macro variables and bond returns has been a long standing puzzle. In this study, we learn that:

1. A simple parsimonious specification with Ψ_t^{π} , Ψ_t^{g} and Ψ_t^{s} helps explain a lot of bond risk premia.
2. $\Psi_t^{\pi,g}$ is consistent with simple RE-heterogeneous model with risk sharing (also affects vol and trade) and explains CP factor.
3. Ψ_t^{s} explain predictability and slope effects to the term structure of vol but it is not spanned or linked to trade: proxy monetary policy?
4. Very large R-square for excess bond returns (between 35% and 40%) and economically significant.
5. Robust to: (a) H1B errors; (b); reverse regressions; (c) lagged disagreement; (e) a host of alternatives ; and (d) stable link over time.